

COMMONWEALTH OF AUSTRALIA



DEPARTMENT OF
HEALTH AND
FAMILY SERVICES

Dose Calculations for Intakes of Ore Dust

by

R. S. O'Brien



Australian Radiation Laboratory

ARL/TR127

Lower Plenty Road, Yallambie, Victoria 3085
Telephone: (03) 9433 2211 Facsimile: (03) 9432 1835
Email: arl.information@health.gov.au
Internet: www.health.gov.au/arl/

AUSTRALIAN RADIATION LABORATORY

Dose Calculations for Intakes of Ore Dust

by

R. S. O'Brien

ARL/TR127
ISSN 0157-1400
AUGUST 1998

LOWER PLENTY ROAD
YALLAMBIE VIC 3085
TELEPHONE: (03) 9433 2211
FAX: (03) 9432 1835

ABSTRACT

This report describes a methodology for calculating the committed effective dose for mixtures of radionuclides, such as those which occur in natural radioactive ores and dusts. The formulae are derived from first principles, with the use of reasonable assumptions concerning the nature and behaviour of the radionuclide mixtures. The calculations are complicated because these 'ores' contain a range of particle sizes, have different degrees of solubility in blood and other body fluids, and also have different biokinetic clearance characteristics from the organs and tissues in the body. The naturally occurring radionuclides also tend to occur in series, i.e. one is produced by the radioactive decay of another 'parent' radionuclide. The formulae derived here can be used, in conjunction with a model such as LUDEP, for calculating total dose resulting from inhalation and/or ingestion of a mixture of radionuclides, and also for deriving annual limits on intake and derived air concentrations for these mixtures.

TABLE OF CONTENTS

1. INTRODUCTION	1
2. THEORY	2
2.1 Total committed effective dose per unit intake	2
2.2 Annual Limit on Intake	5
2.3 Derived Air Concentration	6
3. PRACTICAL CONSIDERATIONS	6
3.1 Calculation of activity fractions for naturally occurring mixtures of radionuclides ..	7
3.2 Dose per unit intake in terms of Bq of ^{238}U or Bq of ^{232}Th	8
3.3 Derivation of results in terms of activity per unit mass of ^{238}U or ^{232}Th	9
3.4 Derivation of the results in terms of alpha activity	10
3.5 Estimation of ^{238}U and ^{232}Th activity from a measurement of total alpha activity ..	12
3.6 Branching chains	12
4. APPLICATIONS	12
4.1 Estimation of dose conversion factors for mixtures of radionuclides	12
4.2 Uranium ore dust and yellowcake	16
4.3 Monazite dust	18
4.4 Sand-blasting operators	20
4.4.1 Calculation of ALI's	20
4.4.2 Limiting specific activity for ilmenite sand-blasting grit	27
4.5 Thorium concentrate	28
5. DISCUSSION AND CONCLUSIONS	29
REFERENCES	31
APPENDICES	33
A.1 Notation	33
A.2 Total committed effective dose for a mixture of radionuclides	34
A.3 Annual Limit on Intake	39
A.4 Calculation of activity fractions	41
A.5 Dose per unit intake in terms of Bq of ^{238}U	44
A.6 ALI in terms of activity per unit mass of ^{238}U or ^{232}Th	46
A.7 Results in terms of alpha activity	47
A.8 ALIs for branching chains	51

TABLES

Table	Caption	Page
Table 4.1-1.	Dose per unit intake factors for the ^{238}U series ($i = 1$)	13
Table 4.1-2.	Dose per unit intake factors for the ^{232}Th series ($i=2$)	14
Table 4.1-3.	Dose per unit intake factors for the ^{235}U series ($i=3$)	14
Table 4.1-4.	Activity fractions for the three naturally occurring radioactive series in different materials.	15
Table 4.1-5	Solubility fractions for some of the materials discussed in this report	15
Table 4.2-1.	Activity fractions, solubility parameters, dose conversion factors and ALIs for Canadian uranium ore dust and yellowcake for a particle size of 5 microns.	17
Table 4.3-1.	Dose conversion factors for each monazite sand sample.	20
Table 4.4.1-1.	^{238}U chain ALI values for particles with an AMAD of $1\ \mu\text{m}$, using data from ICRP Publications 30, 61 and 68.	22
Table 4.4.1-2.	^{232}Th chain ALI values for particles with an AMAD of $1\ \mu\text{m}$, using data from ICRP Publications 30, 61 and 68.	23
Table 4.4.1-3.	^{235}U chain ALI values for particles with an AMAD of $1\ \mu\text{m}$, using data from ICRP Publications 30, 61 and 68.	24
Table 4.4.1-4.	$\sum_j \beta(i,j)/\text{ALI}(i,j)$ for each of the naturally occurring series, using default values from ICRP 30, ICRP 61 and ICRP 68.	25
Table 4.4.2-1.	Total ALIs and limiting values of total specific activity for uranium ore and thorium ore in sand blasting grit, with radon daughters included.	28
Table A.8-1.	Equilibrium factors for the two branches of the Thorium chain.	52
Table A.8-2.	Equilibrium factors for the Thorium chain.	54

FIGURES

Figure	Caption	Page
Figure 1.	Effect of changing solubility on the computed ALI at 5 μm for high temperature calcined yellowcake.	18
Figure 2.	A typical particle size distribution for monazite dust.	19
Figure 3.	ALIs for ilmenite sand blasting grit as a function of Thorium series content.	27

1. INTRODUCTION

Uranium mining and milling, mineral sand mining, and operations such as the clean-up at Maralinga, can produce dusts and product materials which pose a radiological hazard because of the mixtures of radionuclides which they contain. The potential hazard may arise from the presence of uranium, thorium or transuranics, in these materials. A literature survey revealed that, while several procedures are used for assessing measures of dose associated with intakes of mixtures of radionuclides, there is no systematic approach in which the assumptions and limitations of these methods are clearly stated and assessed.

In general, the calculation of committed effective dose or annual limit on intake for a mixture of radionuclides requires a knowledge of the activity fraction for each of the radionuclide series in the mixture and the total activity of the sample. The activity fractions and total activity of a sample may be measured directly. However, the dose per unit intake may also be assessed in terms of the activity of the first member of the series (usually ^{238}U or ^{232}Th), rather than in terms of the total activity of the mixture. Another related problem is the use of measurements of alpha activity of a sample rather than the total activity, or the expression of the activity of the sample in terms of the activity per unit mass of the first member of the series. Finally, not all radioactive series are simple linear decay chains; branches may occur, and the theory needs to take this into account.

This report derives formulae and outlines a methodology for calculating

- committed effective dose (Section 2.1),
- annual limit on intake (Section 2.2), and
- derived air concentration (Section 2.3),

for such mixtures of radionuclides in terms of quantities which can be directly measured, and clearly states the assumptions used in deriving the formulae. This enables the limitations of the approach to be clearly understood.

The notation used in the report is introduced and explained in Appendix A.1. The application of the theory to the solution of various practical problems is discussed in Section 3; these problems include

- the use of activity fractions (Section 3.1),
- the calculation of dose or intake limits in terms of the activity of the first member of a series (Section 3.2),
- the calculation of dose or intake limits in terms of the activity per unit mass of the first member of a series (Section 3.3),
- the calculation of dose or intake limits in terms of the alpha activity of a sample (Section 3.4),
- the estimation of the activity of the first members of the U and Th series from a measurement of the total alpha activity of a sample (Section 3.5), and
- branching radionuclide series (Section 3.6).

The application of the theory to several typical problems involving the dose assessment of intakes of mixtures of radionuclides in the uranium mining and mineral sands industries is also

discussed in detail in Section 4. Some concluding remarks are given in Section 5. Where possible, detailed derivations of the formulae are presented in the Appendices rather than in the main text.

2. THEORY

2.1 Total committed effective dose per unit intake

Consider a dust sample containing a mixture of radionuclides from the three naturally occurring (^{238}U , ^{235}U and ^{232}Th) radioactive decay series. If $A(i,j,s,k,d)$ is the activity of radionuclide species j , in decay series i , solubility class s and biokinetic clearance class k , with particle size d , in this sample, and $D(i,j,s,k,d)$ is the corresponding dose conversion factor (committed effective dose per unit intake), then the committed effective dose $E(i,j,s,k,d)$ resulting from an intake of this single species (of solubility class s , biokinetic clearance class k and particle size d) is given by

$$E(i,j,s,k,d) = D(i,j,s,k,d)A(i,j,s,k,d) \quad (2.1-1)$$

and the total committed effective dose is given by

$$E = \sum_s \sum_k \sum_i \sum_{j=1}^{N(i)} \sum_d D(i,j,s,k,d)A(i,j,s,k,d) \quad (2.1-2)$$

This expression demonstrates the convention (used in the remainder of this report) that, in the present notation, a missing argument indicates that the quantity concerned is calculated by summing over the missing argument; quantities without arguments are calculated by summing over all arguments.

The dose conversion factor (DCF) $D(i,j,s,k,d)$ can be calculated from models (ICRP, 1979; ICRP, 1991) and includes allowance for breathing rate, dissolution in blood and body fluids, and the distribution of inhaled or ingested radionuclides throughout the body by metabolic processes.

The problem with this approach is that it is very difficult to measure $A(i,j,s,k,d)$. What can be measured is the total activity A , and the total activity $A(d)$ of the mixture for each particle size d , where

$$A(d) = \sum_s \sum_k \sum_i \sum_{j=1}^{N(i)} A(i,j,s,k,d) \quad (2.1-3)$$

and the total activity fraction $F(i)$ of each series of radionuclides in the mixture, where

$$F(i) = \frac{A(i)}{A} = \frac{\sum_s \sum_k \sum_{j=1}^{N(i)} \sum_d A(i,j,s,k,d)}{\sum_s \sum_k \sum_t \sum_{j=1}^{N(i)} \sum_d A(i,j,s,k,d)} \quad (2.1-4)$$

The alpha activity $A_\alpha(d)$ for a particular particle size d , and the total alpha activity A_α can also be measured. The first task of the analysis, therefore, is to derive a form of (2.1-3) which allows the measured values of $A(d)$ and $F(n)$ to be used. With this in mind, there are some assumptions which can be made, namely:

- (i) there is only one solubility class and one biokinetic clearance class present,
- (ii) the particle size distribution is the same for all radionuclides in the mixture,
- (iii) the equilibrium between species in the same series is independent of particle size.

It is shown in Appendix A.2 that using these assumptions in equations (2.1-2), (2.1-3) and (2.1-4) leads to the following results (the parameters s and k can be dropped, because of assumption (i)).

The total committed effective dose can now be written as

$$E = \sum_t \sum_{j=1}^{N(i)} \sum_d D(i,j,d) \beta(i,j) \left(\frac{F(i)}{B(i)} \right) A(d) \quad (2.1-5)$$

where

$$B(i) = \sum_{j=1}^{N(i)} \beta(i,j) \quad (2.1-6)$$

and $\beta(i,j)$, which denotes the degree of secular and chemical equilibrium between species in the same radioactive series, is given by

$$A(i,j,d) = \beta(i,j) A(i,1,d) \quad (2.1-7)$$

The factor $\beta(i,j)$ is very important; in some materials particular radionuclides can be partially or completely removed by physical and/or chemical processes. For example, in radioactive dusts, a significant fraction of the $^{222}\text{Rn}/^{220}\text{Rn}$ produced by the decay of $^{226}\text{Ra}/^{224}\text{Ra}$ can escape

from the dust particles by diffusion or recoil processes; in uranium yellowcake, all the radionuclides other than the uranium isotopes ^{238}U , ^{235}U and ^{234}U are removed by chemical processes during the refining of the uranium ore.

For computational purposes (2.1-5) can be written as

$$E = \sum_d A(d) \sum_i \left(\frac{F(i)}{B(i)} \right) \sum_{j=1}^{N(i)} D(i,j,d) \beta(i,j) \quad (2.1-8)$$

The dose conversion factor $D(i,j,d)$ is found by summing over all tissues in the body, using the formula (ICRP 1979, 1991)

$$D(i,j,d) = \sum_T w(T) H(T,i,j,d) \quad (2.1-9)$$

where $w(T)$ ($\equiv w_T$) is the tissue weighting factor (ICRP 1979, 1991), and $H(T,i,j,d)$ ($\equiv H_T(i,j,d)$) is the committed equivalent dose per unit intake in tissue T resulting from the inhalation of material of species j in series i and with particle size d , solubility characteristics s and biokinetic characteristics k . H includes the radiation weighting factor w_R . Therefore the final result can also be written as

$$E = \sum_i \sum_{j=1}^{N(i)} \sum_d \sum_T w(T) H(T,i,j,d) \beta(i,j) \left(\frac{F(i)}{B(i)} \right) A(d) \quad (2.1-10)$$

$A(d)$, $F(i)$, $\beta(i,j)$ and $B(i)$ can be measured; $w(T)$ values are specified by ICRP, and $H(T,i,j,d)$ and $D(i,j,d)$ can be estimated using the ICRP 30 (ICRP, 1979) or ICRP 66 (ICRP, 1991) respiratory tract models.

For an intake of a single radionuclide $j = i = 1$, and $\beta(1,1) = F(1) = B(1) = N(1) = 1$. Substituting these values into equation (2.1-8) gives exactly the result expected for the committed effective dose for an intake of a single radionuclide.

Most of the problems encountered in practice with radionuclide mixtures involve the three naturally occurring decay chains resulting from the decay of ^{238}U , ^{232}Th and ^{235}U . For most of the subsequent discussion, therefore, $i = 1$ refers to the ^{238}U series, $i = 2$ to the ^{232}Th series and $i = 3$ to the ^{235}U series. However the theory developed here can also be used for samples containing mixtures of transuranic radionuclides and fission products, etc.

2.2 Annual Limit on Intake

The Annual Limit on Intake (ALI) is given by the formula

$$ALI = \frac{\text{Dose Limit}}{\text{Dose per Unit Intake}} \quad (2.2-1)$$

Therefore the ALI for a single nuclide is given by

$$ALI(i,j) = \frac{\text{Dose Limit} \cdot \sum_d A(i,j,d)}{\sum_d D(i,j,d)A(i,j,d)} \quad (2.2-2)$$

It is shown in Appendix A.3 that this can be written as

$$ALI(i,j) = \frac{\text{Dose Limit} \cdot \sum_d A(d)}{\sum_d D(i,j,d)A(d)} \quad (2.2-3)$$

For a mixture of radionuclides the dose per unit intake is given by

$$\text{Dose per Unit Intake} = \frac{\text{Total Dose}}{\text{Total Intake}} = \frac{E}{\sum_d A(d)} \quad (2.2-4)$$

Therefore the ALI for a mixture of radionuclides is given by (see Appendix A.3)

$$ALI_{mix} = \frac{\text{Dose Limit} \cdot \sum_d A(d)}{\sum_i \sum_{j=1}^{N(i)} \sum_d D(i,j,d) \beta(i,j) \left(\frac{F(i)}{B(i)} \right) A(d)} \quad (2.2-5)$$

It is also shown in Appendix A.3 that

$$\frac{1}{ALI_{mix}} = \sum_i \sum_{j=1}^{N(i)} \left(\frac{F(i)}{B(i)} \right) \cdot \frac{\beta(i,j)}{ALI(i,j)} \quad (2.2-6)$$

which is equivalent to the standard formula recommended by ICRP for calculating the ALI of a mixture of radionuclides.

2.3 Derived Air Concentration

The derived air concentration (*DAC*) is defined as the concentration in air which, if breathed for the number of hours per year for which the exposure occurs (\mathcal{H}), would give rise to the inhalation of one ALI. Therefore, for a mixture of radionuclides, the *DAC* is given by (using equations (2.2-5) and (2.2-6))

$$DAC_{mix} = \frac{\text{Dose Limit} \cdot \sum_d A(d)}{\mathcal{H} \sum_i \sum_{j=1}^{N(i)} \sum_d D(i,j,d) \beta(i,j) \left(\frac{F(i)}{B(i)} \right) A(d)} \quad (2.3-1)$$

or

$$DAC_{mix} = \frac{1}{\mathcal{H} \sum_i \sum_{j=1}^{N(i)} \left(\frac{F(i)}{B(i)} \right) \cdot \frac{\beta(i,j)}{ALI(i,j)}} \quad (2.3-2)$$

This unit (*DAC*) usually applies to workplace situations, so the exposure time \mathcal{H} is taken as 2000 hours per year. Equation (2.3-2) reduces to the correct formula for a single radionuclide, since in this case (as before) $j = i = 1$ and $\beta(1,1) = F(1) = B(1) = N(1) = 1$.

3. PRACTICAL CONSIDERATIONS

Before applying the theory developed in the previous section, there are several further points to consider:

- (1) To calculate the committed effective dose or ALI for a mixture of radionuclides, the activity fraction for each of the radionuclide series in the mixture has to be known.
- (2) The dose per unit intake of a mixture of radionuclides is frequently expressed in terms of either the activity of ^{238}U or ^{232}Th in the mixture (i.e. Sv / (Bq of ^{238}U) or Sv / (Bq of ^{232}Th)) rather than in terms of the total activity of the mixture.

- (3) In practice, measurements of dust concentrations are usually made by collecting a dust sample on a filter (or screen) and counting the alpha particles emitted when the deposited radionuclides undergo radioactive decay. This means that the formulae already derived have to be expressed in units of alpha activity rather than total activity.
- (4) The activity of ^{238}U or ^{232}Th is sometimes quoted instead of the total activity of the mixture.
- (5) The activity concentration is often expressed as an activity per unit mass of ^{238}U or ^{232}Th , so the results derived here have to include this case.
- (6) Some decay chains are not simple linear chains, so the formulation developed so far has to be adapted to branching chains as well as linear chains.

3.1 Calculation of activity fractions for naturally occurring mixtures of radionuclides

For use in practical applications the activity fractions, $F(n)$, have to be determined. By definition these fractions have to sum to unity, i.e.

$$\sum_i F(i) = 1 \quad (3.1-1)$$

In general, these activity fractions would have to be measured directly. However, for natural uranium, the mass fractions of the uranium isotopes are known to be 99.28%, 0.0058% and 0.71% for ^{238}U , ^{234}U , and ^{235}U respectively (Eisenbud, 1977). It is shown in Appendix A.4 that, for natural uranium, the ratio of ^{235}U activity to ^{238}U activity for natural uranium is 0.0466, or 4.66%. Therefore, for natural uranium, the activity fractions $F(1)$ and $F(3)$ are related by

$$\frac{F(3)}{F(1)} = 0.0466 \frac{B(3)}{B(1)} \quad (3.1-2)$$

If the fraction of activity $F(2)$ in the ^{232}Th series is known, then combining this result with equation (3.1-1) gives

$$F(1) = \frac{1 - F(2)}{1 + 0.0466 \frac{B(3)}{B(1)}} \quad (3.1-3)$$

and

$$F(3) = \frac{0.0466 \frac{B(3)}{B(1)} (1 - F(2))}{1 + 0.0466 \frac{B(3)}{B(1)}} \quad (3.1-4)$$

3.2 Dose per unit intake in terms of Bq of ^{238}U or Bq of ^{232}Th

From the definition of $F(i)$, the total activity in the mixture (A) and the total activity in series i ($A(i)$) are related by

$$A(i) = F(i)A \quad (3.2-1)$$

where $A(i)$ is given by

$$A(i) = \sum_{j=1}^{N(i)} \sum_d A(i,j,d) \quad (3.2-2)$$

It can be shown that (see Appendix A.5), if the total activity due to species 1 in series i is given by $A(i,1)$, then

$$A(i) = B(i)A(i,1) \quad (3.2-3)$$

Therefore

$$A(i,1) = \frac{F(i)}{B(i)}A \quad (3.2-4)$$

Physically this means that one Bq of total activity (i.e. summed over all the nuclides in the mixture) is equivalent to $F(1)/B(1)$ Bq of ^{238}U activity.

This means that the dose per unit intake of ^{238}U is given by

$$\text{Dose per Bq of } ^{238}\text{U} = \frac{E}{A(1,1)} \quad (3.2-5)$$

or, using (3.2-4),

$$\text{Dose per Bq of } ^{238}\text{U} = \frac{EB(1)A}{F(1)} \quad (3.2-6)$$

Therefore the ALI of a mixture of radionuclides is given, in Bq of ^{238}U , by

$$ALI_{\text{mix}} (\text{Bq of } ^{238}\text{U}) = \frac{F(1)}{B(1) \sum_i \sum_{j=1}^{N(i)} \left(\frac{F(i)}{B(i)} \right) \cdot \frac{\beta(i,j)}{ALI(i,j)}} \quad (3.2-7)$$

Therefore

$$ALI_{\text{mix}} (\text{Bq of } ^{238}\text{U}) = \frac{F(1)}{B(1)} \cdot ALI_{\text{mix}} (\text{Bq of dust activity}) \quad (3.2-8)$$

The ALI of the mixture in Bq of ^{232}Th is given by the same formula, with the ratio $F(1)/B(1)$ replaced by $F(2)/B(2)$.

3.3 Derivation of results in terms of activity per unit mass of ^{238}U or ^{232}Th

It is shown in Appendix A.6 that the ALI expressed in grams of ^{238}U is given by

$$ALI_{\text{mix}} (\text{gm of } ^{238}\text{U}) = \frac{m(1,1)F(1)}{\lambda(1,1)B(1)} \cdot ALI_{\text{mix}} (\text{Bq of dust activity}) \quad (3.3-1)$$

or

$$ALI_{\text{mix}} (\text{gm of } ^{238}\text{U}) = \frac{m(1,1)}{\lambda(1,1)} \cdot ALI_{\text{mix}} (\text{Bq of } ^{238}\text{U}) \quad (3.3-2)$$

where $m(1,1)$ is the mass of one atom of ^{238}U , and $\lambda(1,1)$ is the decay coefficient of ^{238}U . Similarly, in terms of the mass of ^{232}Th these results can be written as

$$ALI_{\text{mix}} (\text{gm of } ^{232}\text{Th}) = \frac{m(2,1)F(2)}{\lambda(2,1)B(2)} \cdot ALI_{\text{mix}} (\text{Bq of dust activity}) \quad (3.3-3)$$

or

$$ALI_{mix}(gm \text{ of } ^{232}Th) = \frac{m(2,1)}{\lambda(2,1)} \cdot ALI_{mix}(Bq \text{ of } ^{232}Th) \quad (3.3-4)$$

where $m(2,1)$ is the mass of one atom of ^{232}Th , and $\lambda(2,1)$ is the decay coefficient of ^{232}Th .

3.4 Derivation of the results in terms of alpha activity

Since measurements of dust concentrations are usually made in terms of the total alpha activity deposited on a filter or screen, some way of relating these quantities to the total activity of the mixture of radionuclides is needed. This can be done by writing all previous quantities with a subscript alpha when referring to alpha activity and assuming that the equilibrium factor $\beta_{\alpha}(i,j)$ for an alpha emitter can be written as

$$\beta_{\alpha}(i,j) = \begin{cases} \beta(i,j), & \text{if nuclide } (i,j) \text{ is an } \alpha \text{ emitter} \\ 0, & \text{otherwise.} \end{cases} \quad (3.4-1)$$

The beta emitters have to be included in the calculation of the total dose, because they deliver a dose (which in most cases is much smaller than the dose delivered by a typical alpha emitter). However, because it is relatively easy to measure alpha activity, it is useful to express the ALI in terms of α dps. The dose per α dps is given by

$$\text{Dose per } \alpha\text{dps} = \frac{E}{A_{\alpha}} \quad (3.4-2)$$

where A_{α} is the total alpha activity of the mixture of radionuclides. Note that E is the same dose as before, i.e. the total dose, not the dose delivered by alpha particles. A_{α} is given by

$$A_{\alpha} = \sum_d A_{\alpha}(d) \quad (3.4-3)$$

Therefore the ALI in α dps is given by

$$ALI_{\alpha} = \frac{\text{Dose Limit} \cdot A_{\alpha}}{E} \quad (3.4-4)$$

From the earlier definition (equation (2.2-1)) of the ALI in terms of the total dust activity,

$$ALI = \frac{\text{Dose Limit} \cdot A}{E} \quad (3.4-5)$$

Therefore

$$ALI_{\alpha} = ALI \cdot \frac{A_{\alpha}}{A} \quad (3.4-6)$$

A_{α} can be measured directly. To complete the analysis, it is necessary to establish relationships between A and A_{α} , and between $A(d)$ and $A_{\alpha}(d)$. It is shown in Appendix A.7 that

$$\frac{A_{\alpha}}{A} = \frac{A_{\alpha}(d)}{A(d)} = \sum_i \frac{B_{\alpha}(i)}{B(i)} F(i) \quad (3.4-7)$$

where

$$B_{\alpha}(i) = \sum_{j=1}^{N(i)} \beta_{\alpha}(i,j) \quad (3.4-8)$$

Therefore the total dose is given in terms of alpha activity by

$$E = \frac{\sum_i \sum_{j=1}^{N(i)} \sum_d D(i,j,d) \beta(i,j) \left(\frac{F(i)}{B(i)} \right) A_{\alpha}(d)}{\sum_i \frac{B_{\alpha}(i)}{B(i)} F(i)} \quad (3.4-9)$$

and the ALI (of a mixture) is given in terms of alpha activity by

$$ALI_{\alpha_{\text{mix}}} = ALI_{\text{mix}} \cdot \sum_i \frac{B_{\alpha}(i)}{B(i)} F(i) \quad (3.4-10)$$

Therefore

$$\frac{1}{ALI_{\alpha_{mix}}} = \frac{\sum_i \sum_{j=1}^{N(i)} \left(\frac{F(i)}{B(i)} \right) \cdot \frac{\beta(i,j)}{ALI(i,j)}}{\sum_i \frac{B_{\alpha}(i)}{B(i)} F(i)} \quad (3.4-11)$$

3.5 Estimation of ^{238}U and ^{232}Th activity from a measurement of total alpha activity

Using equations (3.2-4) and (3.4-3) gives

$$A(i,1) = A_{\alpha}(i,1) = \frac{F(i)}{B(i)} \cdot \frac{A_{\alpha}}{\sum_m \frac{B_{\alpha}(m)}{B(m)} F(m)} \quad (3.5-1)$$

Putting $i = 1$ gives the ^{238}U activity, and putting $i = 2$ gives the ^{232}Th activity. This result is only useful if the activity fractions are known or can be measured.

3.6 Branching chains

The ^{232}Th decay chain is not a strictly linear decay chain. However, as is shown in Appendix A.8, a decay chain with branches can be considered as a simple linear decay chain provided the nuclides are labelled carefully and the total equilibrium factors ($B(i)$) are calculated carefully.

4. APPLICATIONS

4.1 Estimation of dose conversion factors for mixtures of radionuclides

This example is included to indicate the data required for use in the main formulae derived in the previous section. The dose per unit intake factors given in Tables 4.1-1, 4.1-2 and 4.1-3 for the nuclides in the three naturally occurring radioactive series were calculated using LUDEP1.1 (Jarvis et al, 1994). Where a value for the dose per unit intake is missing it is considered too small to make a significant contribution to the total dose.

Table 4.1-1. Dose per unit intake factors for the ^{238}U series ($i = 1$).

Nuclide	Bone Seeking Type	Half-life (yrs)	Merge	j	$\beta(1,j)$ (%)	F	M (Sv/Bq)	S
$^{238}\text{U}^*$	S	4.5E+9	N	1	100	2.198E-6	2.122E-6	5.660E-6
	V	4.5E+9	N	1	100	1.167E-7	1.534E-6	5.611E-6
$^{234}\text{U}^*$	S	2.4E+5	N	2	100	2.499E-6	2.653E-6	6.831E-6
	V	2.4E+5	N	2	100	1.329E-7	1.985E-6	6.775E-6
^{234}Pa		2.2E-6		3	100			
$^{230}\text{Th}^*$		7.7E+4	N	4	100		2.289E-5	6.874E-6
$^{226}\text{Ra}^*$	S	1.6E+3	Y	5	100		2.012E-5	
	V	1.6E+3	Y	5	100		1.675E-5	
$^{222}\text{Rn}^*$		1.0E-2		6	100			
$^{218}\text{Po}^*$		5.8E-6	Y	7	100	5.916E-9	7.224E-9	
^{214}Pb		5.1E-5		8	100			
^{214}Bi		3.7E-5		9	100			
$^{214}\text{Po}^*$		8.2E-12	Y	10	100	1.046E-14	1.046E-14	
^{210}Pb		22.0		11	100			
^{210}Bi		1.4E-2		12	100			
$^{210}\text{Po}^*$		3.8E-1	Y	13	100	6.194E-7	2.085E-6	
Total	S					4.833E-5	4.988E-5	4.158E-5
	V					4.051E-5	4.525E-5	3.811E-5

* denotes an alpha emitter.

^{238}U , ^{234}U and ^{235}U tend to become uniformly distributed throughout the bone volume after uptake and deposition in the skeleton (ICRP, 1979), so these nuclides have to be treated as type V bone seekers.

Table 4.1-2. Dose per unit intake factors for the ^{232}Th series ($i=2$)

Nuclide	Bone Seeking Type	Half-life (yrs)	Merge	j	$\beta(2,j)$ (%)	F	M (Sv/Bq)	S
$^{232}\text{Th}^*$		1.4E+10	N	1	100	1.938E-5	5.415E-6	
^{228}Pa				2	100			
^{228}Ac				3	100			
$^{228}\text{Th}^*$		1.9E+0	N	4	100	3.279E-5	2.976E-5	
$^{224}\text{Ra}^*$	S	1.0E-2	Y	5	100	2.506E-6		
	V	1.0E-2	Y	5	100	2.486E-6		
$^{220}\text{Rn}^*$				6	100			
$^{216}\text{Po}^*$		4.0E-9	Y	7	100	8.066E-12	8.068E-12	
^{212}Pb				8	100			
$^{212}\text{Bi}^*$		1.2E-4		9	34	1.506E-8	3.942E-8	
^{212}Bi		1.2E-4		9	66	1.506E-8	3.942E-8	
^{208}Tl				10	34			
$^{212}\text{Po}^*$		1.5E-14		11	66	2.327E-17	2.327E-17	

* denotes an alpha emitter.

Table 4.1.3. Dose per unit intake factors for the ^{235}U series ($i=3$)

Nuclide	Bone Seeking Type	Half-life (yrs)	Merge	j	$\beta(3,j)$ (%)	F	M (Sv/Bq)	S
$^{235}\text{U}^*$	S	7.0E+8	N	1	100	2.309E-6	2.317E-6	6.101E-6
	V	7.0E+8	N	1	100	1.232E-7	1.699E-6	6.050E-6
^{231}U	S	2.9E-3		2	100			
	V	2.9E-3		2	100			
$^{231}\text{Pa}^*$		3.3E+4	Y	3	100		2.340E-5	7.294E-6
^{227}Ac		22		4	100			
$^{227}\text{Th}^*$		5.1E-2	Y	5	100		9.123E-6	1.057E-5
$^{223}\text{Ra}^*$	S	3.1E-2	Y	6	100		5.702E-6	
	V	3.1E-2	Y	6	100		5.664E-6	
$^{219}\text{Rn}^*$		1.2E-7		7	100			
$^{215}\text{Po}^*$		5.6E-11	Y	8	100	8.610E-14	8.610E-14	
^{211}Pb		6.8E-5		9	100			
$^{211}\text{Bi}^*$		4.1E-6		10	100	1.522E-9	1.767E-9	

* denotes an alpha emitter.

The activity fractions of the three radioactivity series for the materials under discussion are tabulated in Table 4.1-4.

Table 4.1-4. Activity fractions for the three naturally occurring radioactive series in different materials.

Material	²³⁸ U F(1)	²³² Th F(2)	²³⁵ U F(3)
uranium ore-dust (no radon loss)	0.962	0.00	0.038
uranium ore-dust (50% radon loss)	0.959	0.00	0.041
non-calcined yellowcake	0.9775	0.00	0.0225
low-temperature calcined yellowcake	0.9775	0.00	0.0225
high-temperature calcined yellowcake	0.9775	0.00	0.0225
monazite dust (no radon loss)	0.0962	0.90	0.0038
monazite dust (50% radon loss)	0.0959	0.90	0.0041

The solubility classification of the different materials are shown in Table 4.1-5.

Table 4.1-5. Solubility fractions for some of the materials discussed in this report.

Material	<i>F</i> <i>Q</i> (1)	<i>M</i> <i>Q</i> (2)	<i>S</i> <i>Q</i> (3)
uranium ore-dust (no radon loss)	0.30	0.00	0.70
uranium ore-dust (50% radon loss)	0.30	0.00	0.70
non-calcined yellowcake	1.00	0.00	0.00
low-temperature calcined yellowcake	1.00	0.00	0.00
high-temperature calcined yellowcake	0.10	0.00	0.90

Since measurements of activity are often made by gross alpha counting, it is desirable to express the various quantities (dose per unit intake of the mixture, ALI) in terms of alpha disintegrations per second (α dps) rather than in becquerels of (total) activity. For example, for the ^{238}U series, one disintegration of ^{238}U corresponds to the emission of eight alpha particles if the members of the ^{238}U series are in secular equilibrium with the ^{238}U (i.e. ore dust).

4.2 Uranium ore dust and yellowcake

Solubility data were supplied (AECL, 1995) for Canadian uranium ore-dust and three types of yellowcake, viz. (1) non-calcined, (2) low temperature calcined, and (3) high temperature calcined yellowcake.

Table 4.2-1 shows the values of the dissolution parameters used in the calculations. The dissolution half-time for the slowly dissolving fraction (T_s) was given as 'several thousand days'. The present calculations assume a value of 5000 days for the half-time of the slowly dissolving fraction of the ore-dust and the high-temperature calcined yellowcake.

U-238, ^{234}U and ^{235}U tend to become uniformly distributed throughout the bone volume after uptake and deposition in the skeleton (ICRP 1979), so these nuclides have to be treated as type V bone seekers. The $D(i,j,d)$ values for each radionuclide and solubility class were calculated using LUDEP1.1 with default values for the occupational case used for the exercise regime, together with an AMAD of $5\ \mu\text{m}$, a GSD of 2.5 and the solubility parameters specified in Table 4.2-1. Where the DCF for a particular nuclide and clearance class could not be calculated because no biokinetic model was available, the value for the nearest clearance class (D, W or Y) for which a DCF could be calculated was substituted. Yellowcake was assumed to contain only ^{238}U , ^{234}U and ^{235}U . Since measurements are often made by gross alpha counting, it is desirable to express the ALI in terms of alpha disintegrations per second rather than becquerels of activity. In this case there are only two alpha particles involved (for yellowcake), since all members of the series other than ^{238}U and ^{234}U are removed during processing of the ore.

The estimated ALIs for the uranium ore dust and yellowcake are also shown in Table 4.2-1.

Table 4.2-1. Activity fractions, solubility parameters, dose conversion factors and ALIs for Canadian uranium ore dust and yellowcake for a particle size of 5 microns.

	Uranium Ore Dust		Yellowcake		
			Non- Calcined	Low T	High T
Percentage of radon (isotopes) lost	0	50	0	0	0
Activity Fractions (%):					
U-238 series	96.5	96.2	97.7	97.7	97.7
U-235 series	3.5	3.8	2.3	2.3	2.3
Solubility Parameters	(1)	(2)	(3)	(4)	(5)
F_r	0.3	0.3	1.0	1.0	0.1
T_r (d)	0.7	0.7	0.14	0.14	0.14
T_s (d)	5000	5000			5000
S_r (d^{-1})	0.99021	0.99021	4.95015	4.95015	4.95015
S_s (d^{-1})	0.000139	0.000139			0.000139
Biokinetic Class	Y	Y	D	D	Y
Total nuclides - B(n)					
U-238 series	14	10	2	2	2
U-235	11	8.5	1	1	1
Total alphas - B_α (n)					
U-238 series	8	6	2	2	2
U-235	7	5.5	1	1	1
CED per Unit Intake (μ Sv per Bq of U-238)	55.9	54.8	.14	.14	10.9
ALI (α dps)	3000	2300	296000	296000	3750

The effect of changing the parameter T_s on the computed ALI for the high temperature calcined yellowcake is shown in Figure 1. The ALI drops from 4100 α dps, for $T_s = 3000$ d, to 3400 α dps for $T_s = 10000$ d.

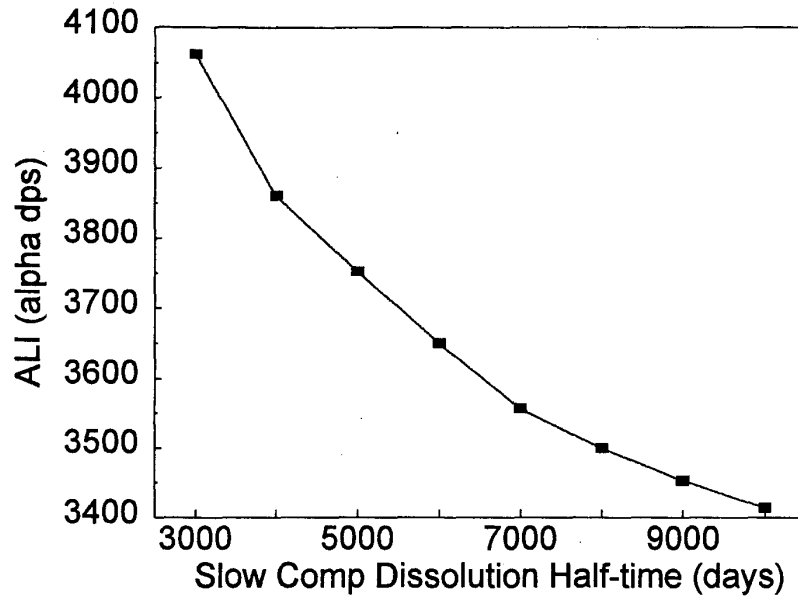


Figure 1. Effect of changing T_s on the computed ALI at $5 \mu\text{m}$ for high temperature calcined yellowcake.

4.3 Monazite dust

The approach outlined here has also been applied to monazite sand particle size distributions measured at a mineral sand facility in Western Australia (Mason and Solomon, 1993; O'Brien and Solomon, 1995). The activity fractions $F(n)$ of the ^{238}U , ^{232}Th and ^{235}U series were taken to be 9.56, 90.0 and 0.44 percent respectively (Mason and Solomon, 1993). The dust samples were collected using Sierra Model 210 and Anderson cascade impactors and the size distributions were determined using both the Twomey and Expectation Maximisation (EM) deconvolution algorithms (Mason and Solomon, 1993). A typical particle size distribution is shown in Figure 2.

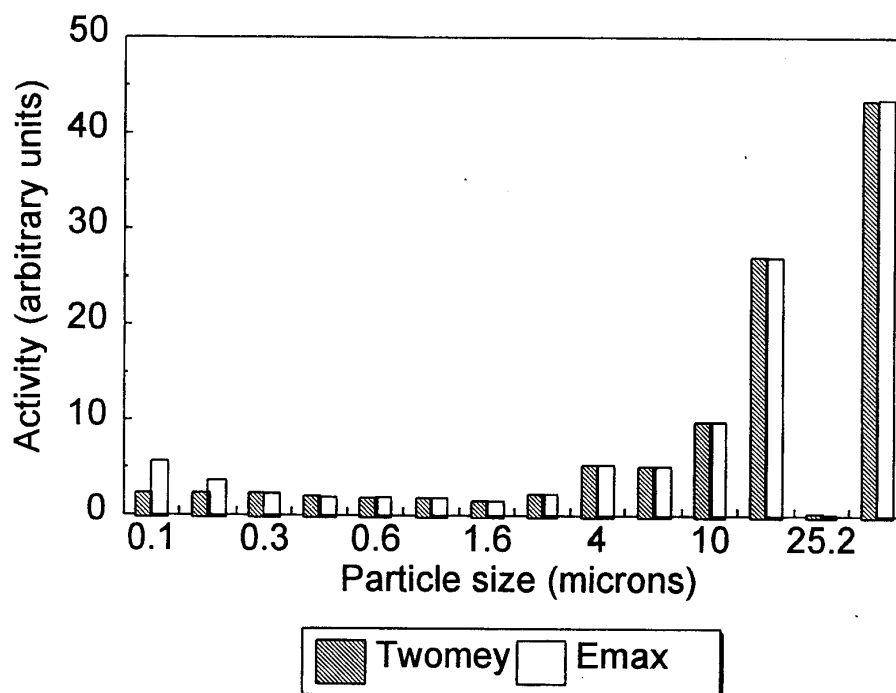


Figure 2. A typical particle size distribution for monazite dust.

The CED per unit intake was calculated for each sample, using the particle size distributions derived from the Twomey and EM de-convolution algorithms. The LUDEP 1.1 calculations were carried out both with and without merging any of the radionuclides (Jarvis et al, 1994) to take account of the in-vivo production of radioactive progeny. The monazite sand can be expected to be very insoluble so the results of the dose calculations are only shown for solubility classes Y (old model) and S (new model). The DCFs (summed over species) were calculated as a function of particle size for both the ICRP 30 and ICRP 66 respiratory tract models. The results of these calculations are shown in Table 4.3-1.

Table 4.3-1. Dose conversion factors for each monazite sand sample.

Measurement	Twomey			Emax		
	ICRP 30	LUDEP 1.1		ICRP 30	LUDEP 1.1	
		un-merged	merged		un-merged	merged
1	2.6E-05	8.45E-07	5.97E-06	2.6E-05	8.98E-07	6.30E-06
2	2.2E-05	3.27E-06	8.55E-06	2.5E-05	3.84E-06	1.02E-05
3	2.4E-05	3.09E-06	7.76E-06	2.7E-05	3.61E-06	9.31E-06
4	2.2E-05	2.84E-06	7.01E-06	2.1E-05	2.94E-06	7.28E-06
5	2.4E-05	3.53E-06	9.34E-06	2.9E-05	4.3E-06	1.16E-05
6	1.7E-05	2.48E-06	6.12E-06	0	0	0
7	2.6E-05	3.37E-06	8.75E-06	2.5E-05	3.52E-06	9.15E-06
8	2.7E-05	3.78E-06	1.00E-05	2.8E-05	3.9E-06	1.03E-05
9	2.4E-05	3.5E-06	9.13E-06	2.6E-05	3.99E-06	1.06E-05
10	2.6E-05	3.34E-06	8.69E-06	2.7E-05	3.78E-06	9.97E-06
11	2.8E-05	4.23E-06	1.17E-05	0	0	0
12	2.9E-05	3.96E-06	1.08E-05	2.6E-05	3.83E-06	1.04E-05
13	2.5E-05	3.46E-06	9.31E-06	2.7E-05	3.85E-06	1.05E-05
14	2.5E-05	3.26E-06	8.43E-06	2.5E-05	3.52E-06	9.20E-06
15	3E-05	4.13E-06	1.11E-05	2.7E-05	3.80E-06	1.01E-05
16	2.3E-05	3.60E-06	9.92E-06	2.6E-05	4.17E-06	1.16E-05
17	2.3E-05	3.02E-06	7.59E-06	2.3E-05	3.26E-06	8.26E-06
18	0	0	0	2.4E-05	3.66E-06	9.63E-06
19	2.3E-05	3.2E-06	8.16E-06	2.6E-05	3.67E-06	9.53E-06
20	2.2E-05	3.12E-06	7.89E-06	2.6E-05	3.80E-06	9.91E-06
21	2.3E-05	2.91E-06	7.27E-06	2.1E-05	2.95E-06	7.36E-06

4.4 Sand-blasting operators

Queensland ilmenite is widely used as sand-blasting grit. Wallace and Leach (1987) quoted specific activities for ^{232}Th and ^{238}U (in the sand-blasting grit) of $240 \pm 80 \text{ Bq kg}^{-1}$ and $95 \pm 15 \text{ Bq kg}^{-1}$ respectively, and an AMAD of 0.9 ± 0.3 microns. In this case, the variables of interest are the ALI of the sand-blasting grit for the operators using the material and the corresponding specific activity for the sand-blasting grit itself, as the values of these variables will affect and/or control the work practices of the operators.

4.4.1 Calculation of ALI's

The starting point for this discussion is equation (2.2-6) for calculating the ALI of a mixture of radionuclides and equation (2.2-3) for the ALI of an individual radionuclide. Equation (2.2-6) expands to

$$\frac{1}{ALI_{mix}} = \left(\frac{F(1)}{B(1)} \right) \sum_{j=1}^{N(1)} \frac{\beta(1,j)}{ALI(1,j)} + \left(\frac{F(2)}{B(2)} \right) \sum_{j=1}^{N(2)} \frac{\beta(2,j)}{ALI(2,j)} + \left(\frac{F(3)}{B(3)} \right) \sum_{j=1}^{N(3)} \frac{\beta(3,j)}{ALI(3,j)} \quad (4.4.1-1)$$

Assuming that the fraction of activity $F(2)$ in the ^{232}Th series is known, and that the sand-blasting grit contains U-238 and U-235 series radionuclides in the same ratio as in 'natural' uranium, equations (3.1-3) and (3.1-4) can be used to eliminate $F(1)$ and $F(3)$. The expression for the ALI of the mixture then becomes

$$\begin{aligned} \frac{1}{ALI_{mix}} = & \left(\frac{1 - F(2)}{B(1) + 0.0466B(3)} \right) \left(\sum_{j=1}^{N(1)} \frac{\beta(1,j)}{ALI(1,j)} + 0.0466 \sum_{j=1}^{N(3)} \frac{\beta(3,j)}{ALI(3,j)} \right) \\ & + \left(\frac{F(2)}{B(2)} \right) \sum_{j=1}^{N(2)} \frac{\beta(2,j)}{ALI(2,j)} \end{aligned} \quad (4.4.1-2)$$

To estimate the ALI for the sand-blasting grit, an AMAD of $1 \mu\text{m}$ was assumed for the particle size. ALIs at $1 \mu\text{m}$ and gut transfer factors (f_i) for individual radionuclides, taken from ICRP Publications 30 (ICRP 1979), 61 (ICRP 1991) and 68 (ICRP 1994b), are shown in Tables 4.4.1-1, 4.4.1-2, and 4.4.1-3.

Table 4.4.1-1. ^{238}U chain ALI values for particles with an AMAD of $1\ \mu\text{m}$, using data from ICRP Publications 30, 61 and 68.

Nuclide	j	$\beta(1,i)$	Class	ICRP 30		ICRP 61		ICRP 68	
				f_i	ALI	f_i	ALI	f_i	ALI
$^{238}\text{U}^*$	1	1.00	D	0.05	5×10^4	0.05	9×10^4	0.02	4×10^4
			W	0.05	3×10^4	0.05	1×10^4	0.02	8×10^3
			Y	2×10^{-3}	3×10^3	2×10^{-3}	600	2×10^{-3}	2700
^{234}Th	2	1.00	W	2×10^{-4}	7×10^6	2×10^{-4}	2×10^6	5×10^{-4}	3×10^6
			Y	2×10^{-4}	6×10^6	2×10^{-4}	2×10^6	2×10^{-4}	3×10^6
^{234}Pa	3	1.00	W	1×10^{-3}	3×10^8	1×10^{-3}	1×10^8	5×10^{-4}	5×10^7
			Y	1×10^{-3}	2×10^8	1×10^{-3}	1×10^8	5×10^{-4}	5×10^7
$^{234}\text{U}^*$	4	1.00	D	0.05	5×10^4	0.05	8×10^4	0.02	4×10^4
			W	0.05	3×10^4	0.05	1×10^4	0.02	7×10^3
			Y	2×10^{-3}	1×10^3	2×10^{-3}	600	2×10^{-3}	2400
$^{230}\text{Th}^*$	5	1.00	W	2×10^{-4}	2×10^2	2×10^{-4}	400	5×10^{-4}	500
			Y	2×10^{-4}	6×10^2	2×10^{-4}	400	2×10^{-4}	1500
$^{226}\text{Ra}^*$	6	1.00	W	0.2	2×10^4	0.2	9000	0.2	1250
$^{222}\text{Rn}^*$	7	1.00							
$^{218}\text{Po}^*$	8	1.00							
^{214}Pb	9	1.00	D	0.2	3×10^7	0.2	1×10^7	0.2	7×10^6
^{214}Bi	10	1.00	D	0.05	3×10^7	0.05	1×10^7	0.05	3×10^6
			W	0.05	3×10^7	0.05	1×10^7	0.05	1.5×10^6
$^{214}\text{Po}^*$	11	1.00							
^{210}Pb	12	1.00	D	0.2	9×10^3	0.2	1×10^4	0.2	2×10^4
^{210}Bi	13	1.00	D	0.05	9×10^6	0.05	9×10^6	0.05	2×10^7
			W	0.05	1×10^6	0.05	4×10^5	0.05	2×10^5
$^{210}\text{Po}^*$	14	1.00	D	0.1	2×10^4	0.1	1×10^4	0.1	3.3×10^4
			W					0.1	6.7×10^4

Table 4.4.1-2. ²³²Th chain ALI values for particles with an AMAD of 1 μm, using data from ICRP Publications 30, 61 and 68.

Nuclide	<i>j</i>	β(2, <i>i</i>)	Class	ICRP 30		ICRP 61		ICRP 68	
				<i>f_i</i>	ALI	<i>f_i</i>	ALI	<i>f_i</i>	ALI
²³² Th*	1	1.00	W	2x10 ⁻⁴	40	2x10 ⁻⁴	90	5x10 ⁻⁴	480
			Y	2x10 ⁻⁴	100	2x10 ⁻⁴	90	2x10 ⁻⁴	900
²²⁸ Ra	2	1.00	W	0.2	4x10 ⁴	0.2	2x10 ⁴	0.2	8x10 ³
²²⁸ Ac	3	1.00	D	1x10 ⁻³	4x10 ⁵	1x10 ⁻³	4x10 ⁵	5x10 ⁻⁴	8x10 ⁵
			W	1x10 ⁻³	1x10 ⁶	1x10 ⁻³	1x10 ⁶	5x10 ⁻⁴	1x10 ⁶
			Y	1x10 ⁻³	2x10 ⁶	1x10 ⁻³	6x10 ⁵	5x10 ⁻⁴	1.5x10 ⁶
²²⁸ Th*	4	1.00	W	2x10 ⁻⁴	400	2x10 ⁻⁴	500	5x10 ⁻⁴	650
			Y	2x10 ⁻⁴	600	2x10 ⁻⁴	200	2x10 ⁻⁴	500
²²⁴ Ra*	5	1.00	W	0.2	2x10 ⁴	0.2	2x10 ⁴	0.2	7x10 ³
²²⁰ Rn*	6	1.00							
²¹⁶ Po*	7	1.00							
²¹² Pb	8	1.00	D	0.2	1x10 ⁶	0.2	5x10 ⁵	0.2	1x10 ⁶
²¹² Bi*	9	0.35	D	0.05	9x10 ⁶	0.05	5x10 ⁶	0.05	2x10 ⁶
			W	0.05	1x10 ⁷	0.05	4x10 ⁶	0.05	7x10 ⁵
²¹² Bi	9	0.65	D	0.05	9x10 ⁶	0.05	5x10 ⁶	0.05	2x10 ⁶
			W	0.05	1x10 ⁷	0.05	4x10 ⁶	0.05	7x10 ⁵
²⁰⁸ Tl	10	0.35							
²¹² Po*	11	0.65							

Table 4.4.1-3. ²³⁵U chain ALI values for particles with an AMAD of 1 μm, using data from ICRP Publications 30, 61 and 68.

Nuclide	<i>j</i>	β(3, <i>i</i>)	Class	ICRP 30		ICRP 61		ICRP 68	
				<i>f_i</i>	ALI	<i>f_i</i>	ALI	<i>f_i</i>	ALI
²³⁵ U*	1	1.00	D	0.05	5x10 ⁴	0.05	8x10 ⁴	0.02	4x10 ⁴
			W	0.05	3x10 ⁴	0.05	1x10 ⁴	0.02	7x10 ³
			Y	2x10 ⁻³	2x10 ³	2x10 ⁻³	600	2x10 ⁻³	2600
²³¹ Th	2	1.00	W	2x10 ⁻⁴	2x10 ⁸	2x10 ⁻⁴	9x10 ⁷	2x10 ⁻⁴	7x10 ⁷
			Y	2x10 ⁻⁴	2x10 ⁸	2x10 ⁻⁴	8x10 ⁷	2x10 ⁻⁴	6x10 ⁷
²³¹ Pa*	3	1.00	W	1x10 ⁻³	60	1x10 ⁻³	100	5x10 ⁻⁴	150
			Y	1x10 ⁻³	100	1x10 ⁻³	100	5x10 ⁻⁴	625
²²⁷ Ac	4	1.00	D	1x10 ⁻³	20	1x10 ⁻³	20	5x10 ⁻⁴	37
			W	1x10 ⁻³	60	1x10 ⁻³	70	5x10 ⁻⁴	95
			Y	1x10 ⁻³	100	1x10 ⁻³	70	5x10 ⁻⁴	300
²²⁷ Th*	5	1.00	W	2x10 ⁻⁴	1x10 ⁴	2x10 ⁻⁴	6000	5x10 ⁻⁴	2600
			Y	2x10 ⁻⁴	1x10 ⁴	2x10 ⁻⁴	5000	2x10 ⁻⁴	2100
²²³ Ra*	6	1.00	W	0.2	3x10 ⁴	0.2	1x10 ⁴	0.2	2900
²¹⁹ Rn*	7	1.00							
²¹⁵ Po*	8	1.00							
²¹¹ Pb	9	1.00	D			0.2	9x10 ⁶	0.2	5x10 ⁶
²¹¹ Bi*	10	1.00							
²⁰⁷ Tl	11	1.00							

For each chain $N(i)$ is equal to the number of nuclides which contribute to the total dose, i.e. $N(1) = 14$, $N(2) = 11$, and $N(3) = 11$. The ²³⁸U and ²³⁵U chains are both linear so, assuming that there is no disequilibrium between the radionuclides, then $\beta(1,j) = 1$ for $j = 1$ to 14, $B(1) = 14$, $\beta(3,j) = 1$ for $j = 1$ to 11, and $B(3) = 11$. For the ²³²Th chain allowance has to be made for the fact that ²¹²Bi is both an alpha emitter (34%) and a beta emitter (66%) and that following the beta decay of ²¹²Bi, ²¹²Po is an alpha emitter. Therefore, if the nuclides are numbered in the sequence shown in Table 4.4.1-2 it is obvious that $\beta(2,j) = 1$ for $j = 1$ to 9, $\beta(2,10) = 0.34$, $\beta(2,11) = 0.66$, and $B(2) = 10$. Therefore the expression for the ALI of the mixture reduces to

$$\frac{1}{ALI_{mix}} = \left(\frac{1 - F(2)}{14.513} \right) \left(\sum_{j=1}^{14} \frac{\beta(1,j)}{ALI(1,j)} + 0.0466 \sum_{j=1}^{11} \frac{\beta(3,j)}{ALI(3,j)} \right) + \left(\frac{F(2)}{10} \right) \sum_{j=1}^{11} \frac{\beta(2,j)}{ALI(2,j)} \quad (4.4.1-3)$$

Table 4.4.1-4 gives values of $\sum_j \beta(i,j)/ALI(i,j)$, computed from the ALI values in the preceding tables (using Class Y data where available, Class W as second choice and Class D where no other data are available).

Table 4.4.1-4. $\sum_j \beta(i,j)/ALI(i,j)$ for each of the naturally occurring series, using default values from ICRP 30, ICRP 61 and ICRP 68.

i	Series	$\sum_j \beta(i,j)/ALI(i,j)$		
		ICRP 30	ICRP 61	ICRP 68
1	²³⁸ U	0.00321	0.00615	0.00233
2	²³² Th	0.01174	0.01616	0.00338
3	²³⁵ U	0.02063	0.02625	0.00614

Using the values for the sums given in Table 4.4.1-4 gives the following expressions for the ALIs:

$$ALI_{mix}(ICRP30) = \frac{10^4}{2.87 + 8.87F(2)}$$

$$ALI_{mix}(ICRP61) = \frac{10^4}{5.08 + 11.08F(2)} \quad (4.4.1-4)$$

$$ALI_{mix}(ICRP68) = \frac{10^4}{1.80 + 1.58F(2)}$$

The calculation of the ALI in α dps proceeds as follows. The expression for ALI_{α} for a mixture of radionuclides is given by

$$ALI_{\alpha_{mix}} = ALI_{mix} \cdot \sum_i \frac{B_{\alpha}(i)}{B(i)} F(i) \quad (4.4.1-5)$$

Now,

$$\sum_i \frac{B_{\alpha}(i)}{B(i)} F(i) = \frac{B_{\alpha}(1)}{B(1)} F(1) + \frac{B_{\alpha}(2)}{B(2)} F(2) + \frac{B_{\alpha}(3)}{B(3)} F(3) \quad (4.4.1-6)$$

Using equations (3.1-3) and (3.1-4), as before, to eliminate $F(1)$ and $F(3)$ gives

$$\sum_i \frac{B_\alpha(i)}{B(i)} F(i) = \left(\frac{B_\alpha(1) + 0.0466B_\alpha(3)}{B(1) + 0.0466B(3)} \right) (1 - F(2)) + \frac{B_\alpha(2)}{B(2)} F(2) \quad (4.4.1-7)$$

Counting the number of alpha-emitters in each series (assuming no disequilibrium) gives $B_\alpha(1) = 8$, $B_\alpha(2) = 6$ and $B_\alpha(3) = 7$. Therefore

$$\sum_i \frac{B_\alpha(i)}{B(i)} F(i) = 0.5737 + 0.0263F(2) \quad (4.4.1-8)$$

Therefore the expressions for the ALIs in terms of α dps are given by the following equations.

$$\begin{aligned} ALI_{\alpha_{mix}}(ICRP30) &= 100 \frac{57.37 + 2.63F(2)}{2.87 + 8.87F(2)} \\ ALI_{\alpha_{mix}}(ICRP61) &= 100 \frac{57.37 + 2.63F(2)}{5.08 + 11.08F(2)} \\ ALI_{\alpha_{mix}}(ICRP68) &= 100 \frac{57.37 + 2.63F(2)}{1.80 + 1.58F(2)} \end{aligned} \quad (4.4.1-9)$$

Once the activity fraction $F(2)$ for the ^{232}Th series radionuclides is known the ALI can be estimated in either Bq of total dust activity, Bq of ^{238}U , or α dps. The ALIs computed from these equations are shown in Figure 3. The open symbols are the ALIs in terms of total activity and the solid symbols represent the ALIs in terms of alpha activity. For each type of curve, the top curve corresponds to ICRP Publication 68 data, the middle curve to ICRP Publication 30 data and the bottom curve to ICRP 61 data. In each case the ALI in α dps is approximately half the ALI in Bq of total activity. The differences between the three data sets (ICRP Publications 30, 61 and 68) are significant and reflect changes and improvements in the data used to calculate the dose per unit intake factors for the individual radionuclides.

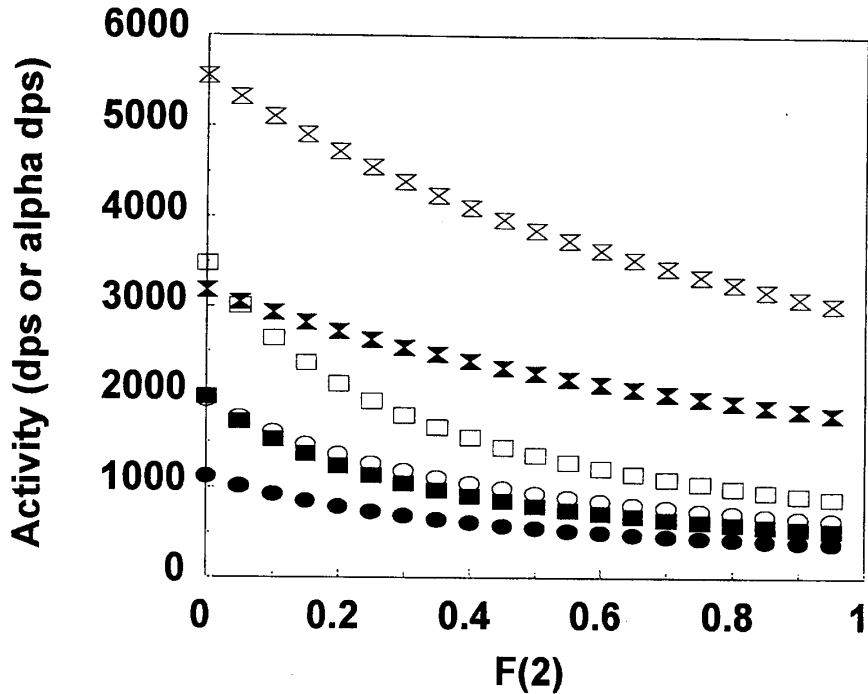


Figure 3. ALI's for ilmenite sand blasting grit as a function of Thorium series content $F(2)$. The open symbols give the ALIs in terms of total activity, using data from ICRP Publication 68 (top), ICRP 30 (middle) and ICRP 61 (bottom). The solid symbols give the corresponding ALIs in units of α dps.

4.4.2 Limiting specific activity for ilmenite sand-blasting grit

The ALI in this case corresponds to an annual dose of 20 mSv. Therefore, the specific activity C (Bq kg^{-1}) in dust which will give an annual dose of 1 mSv (because the operators are members of the public for radiation protection purposes) when M kg of the dust is inhaled in a working year is related to the ALI corresponding to an annual dose of 20 mSv by

$$C = \frac{(ALI)_{20}}{20.M} \text{ Bq kg}^{-1} \quad (4.4.2-1)$$

Assuming a dust concentration in air of 5 mg m^{-3} , a breathing rate of 1.2 m^3 per hour and a working time of 1250 hours per year, in one year an average worker will inhale a total mass

of dust M equal to 7.5×10^{-3} kg. Using the numbers derived earlier, the ALIs and limiting values of specific activity for uranium ore and thorium ore are shown in Table 4.4.2-1.

Table 4.4.2-1. Total ALIs and limiting values of total specific activity for uranium ore and thorium ore in sand blasting grit, with radon daughters included.

	ALI (Bq)	ALI _α (αdps)	C (Bq/kg)	C _α (αdps/kg)
Uranium ore (F(2) = 0)				
ICRP 30	3480	2000	23200	13300
ICRP 61	1970	1130	13100	7530
ICRP 68	5560	3190	37000	21200
Thorium ore (F(2) = 1)				
ICRP 30	850	510	5680	3410
ICRP 61	620	370	4130	2480
ICRP 68	2960	1780	19700	11800

4.5 Thorium concentrate

The ALI of a mixture, expressed in Bq of ^{232}Th , is given by

$$ALI_{\text{mix}}(\text{Bq of } ^{232}\text{Th}) = \frac{F(2)}{B(2)} ALI_{\text{mix}}(\text{Bq of dust activity}) \quad (4.5-1)$$

Also

$$ALI_{\text{mix}}(\text{Bq of } ^{232}\text{Th}) = \frac{F(2)}{B(2)} \cdot \frac{ALI_{\alpha \text{ mix}}}{\sum_i \frac{B_{\alpha}(i)}{B(i)} F(i)} \quad (4.5-2)$$

Therefore, for a mixture containing only Thorium series radionuclides ($F(1) = F(3) = 0$, $F(2) = 1$), and assuming no disequilibrium between the members of the series ($B(2) = 10$), these formulae reduce to

$$ALI_{mix}(Bq \text{ of } ^{232}Th) = \frac{ALI_{mix}(Bq \text{ of dust activity})}{10} \quad (4.5-3)$$

and

$$ALI_{mix}(Bq \text{ of Th-232}) = \frac{ALI_{\alpha mix}}{B_{\alpha}(2)} = \frac{ALI_{\alpha mix}}{6} \quad (4.5-4)$$

5. DISCUSSION AND CONCLUSIONS

The methodology outlined in the previous sections is derived from first principles, using assumptions which are physically and chemically plausible. The results can be applied to any of the naturally occurring radioactive materials containing single radionuclides or mixtures of radionuclides, such as uranium ore dust, thorium ore dust or mineral sand samples. The methodology can also be applied to product materials such as uranium concentrate (yellowcake) and thorium concentrate, and also to mixtures of transuranic radionuclides and fission products. The general derivation of the theory shows that the results only apply to mixtures of radionuclides in which the solubility in body fluids and biokinetic clearance characteristics are the same for all radionuclides in the mixture.

The example on uranium ore dust shows that solubility (i.e. uptake rate) is critically important when determining the dose per unit intake resulting from the inhalation of material containing a mixture of radionuclides. The ICRP 66 respiratory tract model (as implemented by the computer program LUDEP) allows for the solubility of each individual radionuclide to be entered when calculating total effective committed doses and ALIs using the methodology outlined in this report. However, for environmental samples, the solubility of the matrix material is likely to be the controlling factor in the rate at which inhaled radionuclides are dissolved into blood. Furthermore, environmental dust samples are unlikely to contain rapidly dissolving material, as this would have already been removed by dissolution in water over the time scales likely to be involved in the formation and transport of these dusts in the environment. This same argument suggests that it is unlikely that a "standard" value for the solubility of a radioactive dust will apply to all samples of similar material. Therefore, it is important to measure the solubility of dust samples taken from different sites, or use some sample averaging (mixing) technique to ensure that the parameters used in the dose calculations are truly representative of the type of dust sample being assessed.

Once the radionuclides have been taken up by the blood, the biokinetic characteristics of the individual radionuclides in the mixture can be quite different. In addition, for radionuclides which are produced in vivo after uptake, the biokinetic characteristics are likely to depend

strongly on the site of production (e.g. bone or kidney), regardless of the biokinetic characteristics of the parent radionuclide.

The uncertainties in the values of the biokinetic coefficients used in the calculations of the DCFs presented in this report are certainly large enough that most of the DCFs presented in the tables are only accurate to one or two significant figures.

It is a straightforward matter to adapt the formulae developed in this report to intakes of mixtures of radionuclides via the ingestion pathway; for ingestion the dependence on particle size is unimportant.

REFERENCES

- Bailey, M.R., Birchall, A., Cuddihy, R.G., James, A.C., and Roy, M. (1991). *Respiratory Tract Clearance Model for Dosimetry and Bioassay of Inhaled Radionuclides*. Radiation Protection Dosimetry, **38(1/3)**, 153-158.
- Birchall, A., Bailey, M.R., and James, A.C. (1991). *LUDEP: A Lung Dose Evaluation Program*. Radiation Protection Dosimetry, **38(1/3)**, 167-174.
- Eisenbud, M. *Environmental Radioactivity: Second Edition*. New York, Academic Press, 1973.
- ICRP Publication 26 (1977). *Recommendations of the International Commission on Radiological Protection*. Annals of the ICRP, **1(3)**, 1-53.
- ICRP Publication 30 (1979). *Limits for Intakes of Radionuclides by Workers, Part 1*. Annals of the ICRP, **2(3/4)**, 1-116.
- ICRP Publication 56 (1989). *Age-dependent Doses to Members of the Public from Intake of Radionuclides: Part 1*. Annals of the ICRP, **20(2)**, 1-122.
- ICRP Publication 61 (1991). *Annual Limits on Intake of Radionuclides by Workers Based on the 1990 Recommendations*. Annals of the ICRP, **21(4)**, 1-41.
- ICRP Publication 67 (1993). *Age-dependent Doses to Members of the Public from Intake of Radionuclides: Part 2. Ingestion Dose Coefficients*. Annals of the ICRP, **23(3/4)**, 1-167.
- ICRP Publication 66 (1994). *Human Respiratory Tract Model for Radiological Protection*. Annals of the ICRP, **24(1-3)**, 1-482.
- ICRP Publication 68 (1994). *Dose Coefficients for Intakes of Radionuclides by Workers*. Annals of the ICRP, **24(4)**, 1-83.
- James, A.C., Stahlhofen, W., Rudolf, G., Egan, M.J., Nixon, W., Gehr, P., and Briant, J.K. (1991). *The Respiratory Tract Deposition Model Proposed by the ICRP Task Group*. Radiation Protection Dosimetry, **38(1/3)**, 159-165.
- Jarvis, N.S., Birchall, A., James, A.C., Bailey, M.R. and Dorrian, M-D. (1994). *LUDEP 1.1 Personal Computer Program for Calculating Internal Doses Using the New ICRP Respiratory Tract Model*. National Radiological Protection Board, Chilton, Didcot, Oxon OX11 0RQ.
- Mason, G.C. and Solomon, S.B. (1993). *Deconvolution techniques applied to estimation of radiation dose received from inhalation of mineral sands dusts*. Proceedings of the First International Symposium on Radiation Protection in the Mining, Milling and Downstream

Processing of Mineral Sands, held in Bunbury, Western Australia, 18-20 March 1993.
Radiation Protection in Australia, **11(3)**, 133-137.

Wallace, B.J., and Leach, V.A. (1987). *Radiation exposure of sand blasting operators.*
Radiation Protection in Australia, **5(3)**, 63-68.

APPENDICES

A.1 Notation

α dps	alpha decays per second;
AMAD	the activity weighted median aerodynamin diameter of a particle size distribution;
CED	committed effective dose;
DCF	dose conversion factor;
f_i	the gut transfer factor (ICRP, 1979);
GSD	the geometric standard deviation of a particle size distribution;
ICRP	the International Commission on Radiological Protection;
i	the argument denoting a radioactive series $i = 1$ for a radionuclide in the ^{238}U series, $i = 2$ for a radionuclide in the ^{232}Th series, $i = 3$ for a radionuclide in the ^{235}U series;
j	the argument denoting a single radionuclide within a particular series ($j = 1$ for the first member of the series, etc.);
s	the argument denoting solubility class;
k	the argument denoting biokinetic clearance class;
d	the argument denoting particle size;
$A(i,j,s,k,d)$	the activity of radionuclide species j , in decay series i , solubility class s and biokinetic clearance class k , with particle size d , in a sample;
$A(i,j,s,d)$	activity of radionuclide j in series i with particle size d and solubility class s ;
$A(i,j,d)$	activity of radionuclide j in series i with particle size d ;
$A(i,j)$	activity of radionuclide j in series i ;
$A(i)$	total activity of series i radionuclides;
$A(d)$	the total activity of a sample for each particle size d ,
A	the total activity of a sample containing a mixture of radionuclides;
$A_\alpha(d)$	the alpha activity of a sample for a particular particle size d ;
A_α	the total alpha activity of a sample;
ALI	the Annual Limit on Intake for a radionuclide or a mixture of radionuclides, i.e. the activity which, if taken into the body in one year, will give rise to a committed effective dose equal to the prescribed dose limit;
$ALI(i,j)$	the ALI for radionuclide species j in series i ;
ALI_{mix}	the ALI for a specified mixture of radionuclides;
$\beta(i,j)$	the degree of equilibrium (secular and chemical), in terms of activity, between species j in series i and species 1 in series i ;
$\beta_\alpha(i,j)$	the degree of equilibrium (secular and chemical) between species j in series i and species 1 in series i , in terms of activity, for an alpha emitter - $\beta_\alpha(i,j) = 0$ if the radionuclide labelled by i and j is not an alpha emitter;
$B(i)$	the sum of the equilibrium factors $\beta(i,j)$ for series i ;
$B_\alpha(i)$	the sum of the equilibrium factors $\beta_\alpha(i,j)$ for series i ;

DAC	the Derived Air Concentration, i.e. the concentration in air which, if breathed for the number of hours per year for which the exposure occurs (\mathcal{H}), would give rise to the inhalation of one ALI;
$DAC(i,j)$	the derived air concentration for radionuclide species j in series i ;
DAC_{mix}	the derived air concentration for a specified mixture of radionuclides;
$D(i,j,s,k,d)$	the dose conversion factor (committed effective dose per unit intake) for radionuclide species j , in decay series i , with solubility class s , biokinetic clearance class k and particle size d ;
D	dose conversion factor (dose per unit intake);
$E(i,j,s,k,d)$	the committed effective dose resulting from an intake of radionuclide species j in decay series i , solubility class s , biokinetic clearance class k and with particle size d ;
E	committed effective dose;
$F(i)$	the total activity fraction of series i radionuclides in a sample, i.e. the ratio of the total activity of series i radionuclides in the sample to the total activity of the sample;
\mathcal{H}	the number of hours per year for which the exposure occurs;
$H(T,i,j,d)$	(or $H_T(i,j,d)$) the committed equivalent dose per unit intake in tissue T resulting from the inhalation of material of species j in series i and with particle size d , solubility characteristics s and biokinetic characteristics k ;
$\lambda(i,j)$	the decay coefficient of radionuclide species j in series i ;
$m(i,j)$	the mass of one atom of radionuclide species j in series i ;
$M(i,j)$	the total mass of radionuclide species j in series i in a sample;
N_0	Avogadro's number;
$w(T)$	(or w_T) the tissue weighting factor (ICRP 1979, 1991);
F_r	the rapidly dissolving fraction of a sample;
T_r	the dissolution half-time for the rapidly dissolving fraction of a sample;
S_r	the rate coefficient for the rapidly dissolving fraction of a sample ($S_r = \ln(2)/T_r$);
F_s	the slowly dissolving fraction of a sample;
T_s	the dissolution half-time for the slowly dissolving fraction of a sample;
S_s	the rate coefficient for the slowly dissolving fraction of a sample ($S_s = \ln(2)/T_s$).

A.2 Total committed effective dose for a mixture of radionuclides

As explained in the text, if $A(i,j,s,k,d)$ is the activity of radionuclide species j , in decay series i , solubility class s and biokinetic clearance class k , with particle size d , in this sample, and $D(i,j,s,k,d)$ is the corresponding dose conversion factor (committed effective dose per unit intake), then the total committed effective dose resulting from an intake of a mixture of radionuclides is given by

$$E = \sum_x \sum_k \sum_i \sum_{j=1}^{N(i)} \sum_d D(i,j,s,k,d) A(i,j,s,k,d) \quad (\text{A.2-1})$$

The total activity $A(d)$ of the mixture for each particle size d is given by

$$A(d) = \sum_x \sum_k \sum_i \sum_{j=1}^{N(i)} A(i,j,s,k,d) \quad (\text{A.2-2})$$

and the total activity fraction $F(i)$ of each series of radionuclides in the mixture is given by

$$F(i) = \frac{A(i)}{A} = \frac{\sum_x \sum_k \sum_{j=1}^{N(i)} \sum_d A(i,j,s,k,d)}{\sum_x \sum_k \sum_i \sum_{j=1}^{N(i)} \sum_d A(i,j,s,k,d)} \quad (\text{A.2-3})$$

The starting point for the derivation of useful formulae is to assume that

- (i) there is only one solubility class and one biokinetic clearance class present,
- (ii) the particle size distribution is the same for all radionuclides in the mixture,
- (iii) the equilibrium between species in the same series is independent of particle size.

Assumption (i) implies that equations (A.2-1), (A.2-2) and (A.2-3) can be written as

$$E = \sum_i \sum_{j=1}^{N(i)} \sum_d D(i,j,d) A(i,j,d) \quad (\text{A.2-4})$$

and

$$A(d) = \sum_i \sum_{j=1}^{N(i)} A(i,j,d) \quad (\text{A.2-5})$$

and

$$F(i) = \frac{\sum_{j=1}^{N(i)} \sum_d A(i,j,d)}{\sum_i \sum_{j=1}^{N(i)} \sum_d A(i,j,d)} \quad (\text{A.2-6})$$

where it is understood that all these quantities refer to one particular solubility class (s) and one biokinetic clearance class (k).

Assumption (ii) implies that

$$\frac{A(i_1 j_1, d)}{A(i_2 j_2, d)} = f(i_1 j_1; i_2 j_2) \quad (\text{A.2-7})$$

where f is some unknown function. This can only be true if

$$A(i,j,d) = g(i,j)a(d) \quad (\text{A.2-8})$$

for some unknown functions $g(i,j)$ and $a(d)$. Therefore

$$A(d) = \sum_i \sum_{j=1}^{N(i)} g(i,j)a(d) = G \cdot a(d) \quad (\text{A.2-9})$$

where G is an unknown function given by

$$G = \sum_i \sum_{j=1}^{N(i)} g(i,j) \quad (\text{A.2-10})$$

Therefore

$$a(d) = \frac{A(d)}{G} \quad (\text{A.2-11})$$

Also

$$F(i) = \frac{\sum_{j=1}^{N(i)} g(i,j)}{G} \quad (\text{A.2-12})$$

and

$$E = \sum_i \sum_{j=1}^{N(i)} \sum_d D(i,j,d) g(i,j) a(d) \quad (\text{A.2-13})$$

This last result can be re-written as

$$E = \frac{\sum_i \sum_{j=1}^{N(i)} \sum_d D(i,j,d) g(i,j) A(d)}{G} \quad (\text{A.2-14})$$

The degree of secular and chemical equilibrium between species in the same radioactive series can be expressed in terms of an equilibrium factor $\beta(i,j,d)$ which is defined by

$$A(i,j,d) = \beta(i,j,d) A(i,1,d) \quad (\text{A.2-15})$$

Assumption (iii) implies that $\beta(i,j,d) = \beta(i,j)$, where $\beta(i,1) = 1$ for all i . Therefore

$$A(i,j,d) = \beta(i,j) A(i,1,d) \quad (\text{A.2-16})$$

But from (A.2-8) we also have

$$A(i,j,d) = g(i,j) a(d), \quad A(i,1,d) = g(i,1) a(d) \quad (\text{A.2-17})$$

Therefore

$$g(i,j) = \beta(i,j) g(i,1) \quad (\text{A.2-18})$$

This means that (A.2-12) becomes

$$F(i) = \frac{g(i,1) \sum_{j=1}^{N(i)} \beta(i,j)}{G} = \frac{g(i,1)B(i)}{G} \quad (\text{A.2-19})$$

where

$$B(i) = \sum_{j=1}^{N(i)} \beta(i,j) \quad (\text{A.2-20})$$

and

$$E = \frac{\sum_i \sum_{j=1}^{N(i)} \sum_d D(i,j,d) \beta(i,j) g(i,1) A(d)}{G} \quad (\text{A.2-21})$$

Solving (A.2-19) for $g(i,1)$ and substituting into (A.2-21) gives

$$E = \sum_i \sum_{j=1}^{N(i)} \sum_d D(i,j,d) \beta(i,j) \left(\frac{F(i)}{B(i)} \right) A(d) \quad (\text{A.2-22})$$

The dose conversion factor $D(i,j,d)$ is found by summing over all tissues, using the formula (ICRP 1979, 1991)

$$D(i,j,d) = \sum_T w(T) H(T,i,j,d) \quad (\text{A.2-23})$$

where $w(T)$ is the tissue weighting factor for tissue T , and $H(T,i,j,d)$ is the committed equivalent dose per unit intake in tissue T resulting from the inhalation of material (with solubility characteristics s and biokinetic characteristics k) of species j in series i and with particle size d . Therefore (A.2-22) can be written as

$$E = \sum_i \sum_{j=1}^{N(i)} \sum_d \sum_T w(T) H(T,i,j,d) \beta(i,j) \left(\frac{F(i)}{B(j)} \right) A(d) \quad (\text{A.2-24})$$

$A(d)$, $F(i)$, $\beta(i,j)$ and $B(i)$ can be measured; the $w(T)$ values are specified by ICRP, and $H(T,i,j,d)$ and $D(i,j,d)$ can be estimated using the ICRP 30 or ICRP 66 respiratory tract models.

A.3 Annual Limit on Intake

The Annual Limit on Intake (ALI) is given by the formula

$$ALI = \frac{\text{Dose Limit}}{\text{Dose per Unit Intake}} \quad (\text{A.3-1})$$

The dose per unit intake for a single nuclide is given (in the present notation) by

$$\text{Dose per unit intake } (i,j) = \frac{\sum_d D(i,j,d)A(i,j,d)}{\sum_d A(i,j,d)} \quad (\text{A.3-2})$$

Therefore the ALI for a single nuclide is given by

$$ALI(i,j) = \frac{\text{Dose Limit} \cdot \sum_d A(i,j,d)}{\sum_d D(i,j,d)A(i,j,d)} \quad (\text{A.3-3})$$

Using (A.2-17) and (A.2-11), this becomes

$$ALI(i,j) = \frac{\text{Dose Limit} \cdot \sum_d A(d)}{\sum_d D(i,j,d)A(d)} \quad (\text{A.3-4})$$

For a mixture of radionuclides the dose per unit intake is given by

$$\text{Dose per Unit Intake} = \frac{E}{\sum_d A(d)} \quad (\text{A.3-5})$$

Substituting for E from (A.2-22), and substituting the result into (A.3-1) gives the ALI for a mixture of radionuclides as

$$ALI_{mix} = \frac{Dose\ Limit \cdot \sum_d A(d)}{\sum_i \sum_{j=1}^{N(i)} \sum_d D(i,j,d) \beta(i,j) \left(\frac{F(i)}{B(i)} \right) A(d)} \quad (A.3-6)$$

Inverting this equation gives

$$\frac{1}{ALI_{mix}} = \frac{\sum_i \sum_{j=1}^{N(i)} \sum_d D(i,j,d) \beta(i,j) \left(\frac{F(i)}{B(i)} \right) A(d)}{Dose\ Limit \cdot \sum_d A(d)} \quad (A.3-7)$$

In terms of the ALI's of the individual radionuclides (equation (A.3-4)) this can be written as

$$\frac{1}{ALI_{mix}} = \sum_i \sum_{j=1}^{N(i)} \left(\frac{F(i)}{B(i)} \right) \cdot \frac{\beta(i,j)}{ALI(i,j)} \quad (A.3-8)$$

which is equivalent to the standard formula recommended by ICRP for calculating the ALI of a mixture.

For the case of a mixture of radionuclides which are completely independent of each other, the standard ICRP formula is obtained by noting that, in this case, $j = 1$ and $N(i) = \beta(i,1) = B(i) = 1$, for all i . Therefore in this case equation (A.3-8) reduces to

$$\frac{1}{ALI_{mix}} = \sum_i \frac{F(i)}{ALI(i,1)} \quad (92)$$

where $F(i)$ is the fractional concentration of the i -th member of the mixture. Since this result can be derived independently, this is a useful check on the algebra.

A.4 Calculation of activity fractions

For use in practical applications, the $F(i)$ have to be determined. For natural uranium, the mass fractions of the uranium isotopes are 99.28%, 0.0058% and 0.71% for ^{238}U , ^{234}U , and ^{235}U respectively (Eisenbud, 1977).

In the present notation the ratio $R_U = \text{Activity}(^{235}\text{U}) : (\text{total uranium activity})$ is given by

$$R_U = \frac{\sum_d A(3,1,d)}{\sum_d (A(1,1,d) + A(1,2,d) + A(3,1,d))} \quad (\text{A.4-1})$$

Using (A.2-16), this becomes

$$R_U = \frac{\sum_d \beta(3,1)A(3,1,d)}{\sum_d (\beta(1,1)A(1,1,d) + \beta(1,2)A(1,2,d) + \beta(3,1)A(3,1,d))} \quad (\text{A.4-2})$$

By definition $\beta(i,1) = 1$. ^{234}U is in equilibrium with its parent ^{238}U , so $\beta(1,2) = 1$. Therefore

$$R_U = \frac{\sum_d A(3,1,d)}{\sum_d (2A(1,1,d) + A(3,1,d))} \quad (\text{A.4-3})$$

Carrying out the summations over d gives

$$R_U = \frac{A(3,1)}{2A(1,1) + A(3,1)} \quad (\text{A.4-4})$$

In the present notation the activity of an individual nuclide is given by $A(i,j) = \lambda(i,j)n(i,j)$. This means that the ratio R_U is given by

$$R_U = \frac{\lambda(3,1)n(3,1)}{2\lambda(1,1)n(1,1) + \lambda(3,1)n(3,1)} \quad (\text{A.4-5})$$

where the n 's denote the number of atoms of the two radionuclides, and the λ 's denote the radioactive decay constants. The half-lives are 4.5 x 10⁹ and 7.0 x 10⁸ years for ^{238}U and ^{235}U respectively, and the decay constant is given by $\lambda(i,j) = \log_e(2)/\tau$, so (A.4-5) becomes

$$R_U = \frac{4.5n(3,1)}{1.4n(1,1) + 4.5n(3,1)} \quad (\text{A.4-6})$$

The masses of ^{235}U and ^{238}U are in the ratio (Eisenbud, 1977)

$$\frac{M(3,1)}{M(1,1)} = \frac{0.0071}{0.9928} = 7.1515 \times 10^{-3} \quad (\text{A.4-7})$$

where $M(1,1)$ is the total mass of ^{238}U in the ore-dust. Since $M(i,j) = n(i,j)m(i,j)$, where $m(i,j)$ is the mass of one atom of species j in series i , this gives

$$\frac{n(3,1)m(3,1)}{n(1,1)m(1,1)} = 7.1515 \times 10^{-3} \quad (\text{A.4-8})$$

For 1 mole

$$m(1,1)N_0 = 238, \quad m(3,1)N_0 = 235 \quad (\text{A.4-9})$$

where N_0 is Avogadro's number (the number of atoms in 1 mole). Therefore (for natural uranium)

$$\frac{235n(3,1)}{238n(1,1)} = 7.1515 \times 10^{-3} \quad (\text{A.4-10})$$

Therefore

$$n(3,1) = 7.243 \times 10^{-3} n(1,1) \quad (\text{A.4-11})$$

Substituting this result into (A.4-6) gives

$$R_U = 0.02275 \quad (\text{A.4-12})$$

Therefore (A.4-2) becomes

$$\frac{\sum_d A(3,1,d)}{\sum_d (A(1,1,d) + A(1,2,d) + A(3,1,d))} = 0.02275 \quad (\text{A.4-13})$$

This can be written as

$$\frac{\sum_d g(3,1)a(d)}{\sum_d (g(1,1)a(d) + g(1,2)a(d) + g(3,1)a(d))} = 0.02275 \quad (\text{A.4-14})$$

Summing over d and cancelling gives

$$\frac{g(3,1)}{g(1,1) + g(1,2) + g(3,1)} = 0.02275 \quad (\text{A.4-15})$$

Using equation (A.2-18) to eliminate $g(1,2)$, and noting that, for natural uranium, ^{238}U and ^{234}U are in secular equilibrium, so that $\beta(1,2) = 1$, gives

$$g(3,1) = 0.0466g(1,1) \quad (\text{A.4-16})$$

For a single particle size

$$R_U(d) = \frac{A(3,1,d)}{(2A(1,1,d) + A(3,1,d))} \quad (\text{A.4-17})$$

which leads to the same result.

This implies that the ratio of ^{235}U activity to ^{238}U activity for natural uranium is 0.0466, or 4.66%. Therefore the ratio of total ^{235}U series activity to total ^{238}U series activity is given by (using (A.2-19))

$$\frac{F(3)}{F(1)} = \frac{B(3)g(3,1)}{B(1)g(1,1)} = 0.0466X = Z \quad (\text{A.4-18})$$

where

$$X = \frac{\sum_{j=1}^{N(3)} \beta(3,j)}{\sum_{j=1}^{N(1)} \beta(1,j)} = \frac{B(3)}{B(1)} \quad (\text{A.4-19})$$

The total activity of all three series can be determined from the basic relation $F(1) + F(2) + F(3) = 1$. If $F(1)$ is known, then $F(3)$ and $F(2)$ can be determined. If the fraction of activity $F(2)$ in the ^{232}Th series is known, then

$$F(1) + F(3) = 1 - F(2) = Y \quad (\text{A.4-20})$$

Combining these results gives

$$F(1) = \frac{Y}{1 + Z} = \frac{1 - F(2)}{1 + 0.0466 \frac{B(3)}{B(1)}} \quad (\text{A.4-21})$$

and

$$F(3) = \frac{Z.Y}{1 + Z} = \frac{0.0466 \frac{B(3)}{B(1)} (1 - F(2))}{1 + 0.0466 \frac{B(3)}{B(1)}} \quad (\text{A.4-22})$$

A.5 Dose per unit intake in terms of Bq of ^{238}U

From the definition of $F(i)$, the total activity A and the total activity $A(i)$ in series i are related by

$$A(i) = F(i)A \quad (\text{A.5-1})$$

where $A(i)$ is given by

$$A(i) = \sum_{j=1}^{N(i)} \sum_d A(i,j,d) \quad (\text{A.5-2})$$

From (A.2-16) and (A.2-20) we have

$$A(i) = \sum_{j=1}^{N(i)} \sum_d \beta(i,j) A(i,1,d) = B(i) \sum_d A(i,1,d) \quad (\text{A.5-3})$$

The total activity $A(i,1)$ due to species 1 in series i is given by

$$A(i,1) = \sum_d A(i,1,d) \quad (\text{A.5-4})$$

Therefore

$$A(i) = B(i)A(i,1) \quad (\text{A.5-5})$$

Equating (A.5-1) and (A.5-5) and solving for $A(i,1)$ gives

$$A(i,1) = \frac{F(i)}{B(i)} A \quad (\text{A.5-6})$$

Physically this means that one Bq of total activity (i.e. summed over all the nuclides in the mixture) is equivalent to $F(1)/B(1)$ Bq of ^{238}U activity (denote the three series by $i = 1 \rightarrow ^{238}\text{U}$ series, $i = 2 \rightarrow ^{232}\text{Th}$ series and $i = 3 \rightarrow ^{235}\text{U}$ series).

This means that the dose per unit intake of ^{238}U is given by

$$\text{Dose per Bq of } ^{238}\text{U} = \frac{E}{A(1,1)} \quad (\text{A.5-7})$$

Using (4.5-6) gives

$$\text{Dose per Bq of } ^{238}\text{U} = \frac{EB(1)}{F(1) \sum_d A(d)} \quad (\text{A.5-8})$$

Using (A.2-22) for E and substituting the resulting equation into (A.3-1) shows that the ALI in Bq of ^{238}U is given by

$$\text{ALI(Bq of } ^{238}\text{U)} = \frac{\text{Dose Limit} \cdot F(1) \cdot \sum_d A(d)}{B(1) \sum_i \sum_{j=1}^{N(i)} \sum_d D(i,j,d) \beta(i,j) \left(\frac{F(i)}{B(i)} \right) A(d)} \quad (\text{A.5-9})$$

Comparing this result with (A.3-6) shows that

$$ALI(Bq \text{ of } ^{238}\text{U}) = \frac{F(1)}{B(1)} ALI(Bq \text{ of dust activity}) \quad (\text{A.5-10})$$

A.6 ALI in terms of activity per unit mass of ^{238}U or ^{232}Th

The total mass, $M(1,1)$, of ^{238}U in a sample of ore dust is given by

$$M(1,1) = m(1,1)n(1,1) \quad (\text{A.6-1})$$

where $n(1,1)$ is the total number of atoms of ^{238}U in the sample and $m(1,1)$ is the mass of one atom of ^{238}U . The ALI of the mixture in gm of ^{238}U is given by

$$ALI_{\text{mix}}(\text{gm of } ^{238}\text{U}) = \frac{\text{Dose Limit}}{\text{Dose per gm of } ^{238}\text{U}} \quad (\text{A.6-2})$$

Therefore

$$ALI_{\text{mix}}(\text{gm of } ^{238}\text{U}) = \frac{\text{Dose Limit} \cdot \text{mass of } ^{238}\text{U}}{E} \quad (\text{A.6-3})$$

Therefore

$$ALI_{\text{mix}}(\text{gm of } ^{238}\text{U}) = \frac{\text{Dose Limit} \cdot m(1,1) \cdot n(1,1)}{E} \quad (\text{A.6-4})$$

The total activity of ^{238}U is given by $A(1,1) = \lambda(1,1)n(1,1)$, where $\lambda(1,1)$ is the decay coefficient of ^{238}U . Therefore

$$ALI_{\text{mix}}(\text{gm of } ^{238}\text{U}) = \frac{\text{Dose Limit} \cdot m(1,1) \cdot A(1,1)}{E \cdot \lambda(1,1)} \quad (\text{A.6-5})$$

Using (A.5-6) to replace $A(1,1)$ gives

$$ALI_{\text{mix}}(\text{gm of } ^{238}\text{U}) = \frac{\text{Dose Limit} \cdot m(1,1) \cdot A \cdot F(1)}{E \cdot \lambda(1,1)B(1)} \quad (\text{A.6-6})$$

Comparing this result with (A.3-6) gives

$$ALI_{mix}(gm \text{ of } ^{238}U) = \frac{m(1,1)F(1)}{\lambda(1,1)B(1)} \cdot ALI_{mix}(Bq \text{ of dust activity}) \quad (A.6-7)$$

or, from (A.5-10),

$$ALI_{mix}(gm \text{ of } ^{238}U) = \frac{m(1,1)}{\lambda(1,1)} \cdot ALI_{mix}(Bq \text{ of } ^{238}U) \quad (A.6-8)$$

Similarly, in terms of the mass of ^{232}Th these results can be written as

$$ALI_{mix}(gm \text{ of } ^{232}Th) = \frac{m(2,1)F(2)}{\lambda(2,1)B(2)} \cdot ALI_{mix}(Bq \text{ of dust activity}) \quad (A.6-9)$$

or

$$ALI_{mix}(gm \text{ of } ^{232}Th) = \frac{m(2,1)}{\lambda(2,1)} \cdot ALI_{mix}(Bq \text{ of } ^{232}Th) \quad (A.6-10)$$

A.7 Results in terms of alpha activity

Since measurements of dust concentrations are usually made in terms of the total alpha activity deposited on a filter or screen we need some way of relating these quantities to the total activity of the mixture of radionuclides. This can be done by writing all previous quantities with a subscript alpha when referring to alpha activity and assuming that

$$\beta_{\alpha}(i,j) = \begin{cases} \beta(i,j), & \text{if nuclide } (i,j) \text{ is an } \alpha \text{ emitter} \\ 0, & \text{otherwise} \end{cases} \quad (A.7-1)$$

The beta emitters have to be included in the calculation of the total dose, because they deliver a dose (which in most cases is much smaller than the dose delivered by a typical alpha emitter). However, because it is relatively easy to measure alpha activity, it is useful to express the ALI in terms of α dps. The dose per α dps is given by

$$Dose \text{ per } \alpha dps = \frac{E}{A_{\alpha}} \quad (A.7-2)$$

where A_α is the total alpha activity of the mixture of radionuclides. Note that E is the same dose as before, i.e. the total dose, not the dose delivered by alpha particles. A_α is given by

$$A_\alpha = \sum_d A_\alpha(d) \quad (\text{A.7-3})$$

Therefore the ALI of the mixture in α dps is given by

$$ALI_{\alpha \text{ mix}} = \frac{\text{Dose Limit} \cdot A_\alpha}{E} \quad (\text{A.7-4})$$

From the earlier definition of the ALI in terms of the total dust activity we have

$$ALI = \frac{\text{Dose Limit} \cdot A}{E} \quad (\text{A.7-5})$$

Therefore

$$ALI_\alpha = ALI \cdot \frac{A_\alpha}{A} \quad (\text{A.7-6})$$

where the subscript 'mix' has been dropped for clarity.

A_α can be measured directly. To complete the analysis, it is necessary to establish a relationship between $A(d)$ and $A_\alpha(d)$. To do this write

$$\frac{A_\alpha}{A} = \frac{\sum_d \sum_i \sum_{j=1}^{N(i)} A_\alpha(i,j,d)}{\sum_d \sum_i \sum_{j=1}^{N(i)} A(i,j,d)} \quad (\text{A.7-7})$$

But

$$A_\alpha(i,j,d) = \beta_\alpha(i,j) A(i,1,d) \quad (\text{A.7-8})$$

and $A(i,1,d)$ is given by (A.2-8). Therefore

$$\frac{A_\alpha}{A} = \frac{\sum_d \sum_i \sum_{j=1}^{N(i)} \beta_\alpha(i,j) g(i,1) a(d)}{\sum_d \sum_i \sum_{j=1}^{N(i)} \beta(i,j) g(i,1) a(d)} = \frac{\sum_i B_\alpha(i) g(i,1)}{\sum_i B(i) g(i,1)} \quad (\text{A.7-9})$$

where

$$B_\alpha(i) = \sum_{j=1}^{N(i)} \beta_\alpha(i,j) \quad (\text{A.7-10})$$

Solving (A.2-19) for $g(i,1)$ and substituting the result in (A.7-9) gives

$$\frac{A_\alpha}{A} = \frac{\sum_i B_\alpha(i) \frac{F(i)G}{B(i)}}{\sum_i B(i) \frac{F(i)G}{B(i)}} = \frac{\sum_i \frac{B_\alpha(i)}{B(i)} F(i)}{\sum_i F(i)} = \sum_i \frac{B_\alpha(i)}{B(i)} F(i) \quad (\text{A.7-11})$$

The ratio of alpha activity to total activity at a particular particle size d is given by

$$\frac{A_\alpha(d)}{A(d)} = \frac{\sum_i \sum_{j=1}^{N(i)} A_\alpha(i,j,d)}{\sum_i \sum_{j=1}^{N(i)} A(i,j,d)} \quad (\text{A.7-12})$$

Using (A.7-8) and (A.2-8) gives

$$\frac{A_\alpha(d)}{A(d)} = \frac{\sum_i \sum_{j=1}^{N(i)} \beta_\alpha(i,j) g(i,1) a(d)}{\sum_i \sum_{j=1}^{N(i)} \beta(i,j) g(i,1) a(d)} = \frac{\sum_i B_\alpha(i) g(i,1)}{\sum_i B(i) g(i,1)} \quad (\text{A.7-13})$$

Solving (A.2-19) for $g(i,1)$ and substituting the result in (A.7-13) gives

$$\frac{A_\alpha(d)}{A(d)} = \frac{\sum_i B_\alpha(i) \frac{F(i)G}{B(i)}}{\sum_i B(i) \frac{F(i)G}{B(i)}} = \sum_i \frac{B_\alpha(i)}{B(i)} F(i) \quad (\text{A.7-14})$$

or

$$A(d) = \frac{A_\alpha(d)}{\sum_i \frac{B_\alpha(i)}{B(i)} F(i)} \quad (\text{A.7-15})$$

Therefore the ALI in terms of alpha activity is given by

$$ALI_\alpha = ALI \cdot \sum_i \frac{B_\alpha(i)}{B(i)} F(i) \quad (\text{A.7-16})$$

and the total dose in terms of alpha activity is given by

$$E = \frac{\sum_i \sum_{j=1}^{N(i)} \sum_d D(i,j,d) \beta(i,j) \left(\frac{F(i)}{B(i)} \right) A_\alpha(d)}{\sum_i \frac{B_\alpha(i)}{B(i)} F(i)} \quad (\text{A.7-17})$$

Now,

$$\sum_i \frac{B_\alpha(i)}{B(i)} F(i) = \frac{B_\alpha(1)}{B(1)} F(1) + \frac{B_\alpha(2)}{B(2)} F(2) + \frac{B_\alpha(3)}{B(3)} F(3) \quad (\text{A.7-18})$$

Using eqns. (A.4-21) and (A.4-22) for the activity fractions $F(1)$ and $F(3)$ gives

$$\sum_i \frac{B_\alpha(i)}{B(i)} F(i) = \frac{(B_\alpha(1) + 0.0466B_\alpha(3))(1 - F(2))}{B(1) + 0.0466B(3)} + \frac{B_\alpha(2)}{B(2)} F(2) \quad (\text{A.7-19})$$

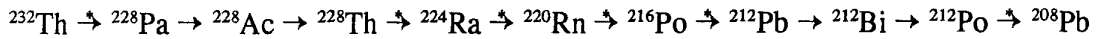
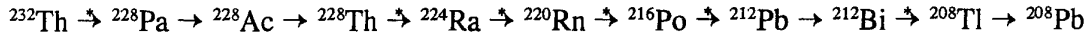
A.8 ALIs for branching chains

The ^{232}Th chain is not a strictly linear chain. The interesting part of the chain for the purposes of the current discussion is shown below. One way of analysing this situation is to assume that the Thorium chain can be broken up into two linear chains, one of which proceeds via the alpha decay of ^{212}Bi and beta decay of ^{208}Tl to ^{208}Pb while the other proceeds via beta decay of ^{212}Bi and alpha decay of ^{212}Po to ^{208}Pb . Obviously each of these chains contains the same number of nuclides, but some care is needed in counting the total number of alpha particles, i.e. in computing the equilibrium factors $\beta(i,j)$ and $\beta_{\alpha}(i,j)$ and the totals $B(i)$ and $B_{\alpha}(i)$.

At this point, a minor change in notation is needed. If the ^{208}Tl branch is labelled chain 1 ($i=1$) and the ^{212}Po branch is labelled chain 2 ($i=2$), then for a sample containing Thorium series nuclides only the total dose is given by

$$ALI_{\alpha} = \frac{\text{Dose Limit} \cdot A_{\alpha}}{\sum_{i=1}^2 \sum_{j=1}^{N(i)} \sum_d D(i,j,d) \beta(i,j) \left(\frac{F(i)}{B(i)} \right) A(d)} \quad (\text{A.8-1})$$

The $i=1$ and $i=2$ branches contain the transitions (\rightarrow denotes alpha decay, \rightarrow denotes beta decay)



The $i=1$ branch contains 6 alpha decays, and the $i=2$ branch contains 6 alpha decays. The equilibrium factors for the different members of the two chains are set out in Table A.8-1.

Table A.8-1. Equilibrium factors for the two branches of the Thorium chain.

Nuclide	i	j	$\beta(1,j)$	$\beta_\alpha(1,j)$	$\beta(2,j)$	$\beta_\alpha(2,j)$
$^{232}\text{Th}^*$	1,2	1	0.34	0.34	0.66	0.66
^{228}Ra	1,2	2	0.34	0.00	0.66	0.00
^{228}Ac	1,2	3	0.34	0.00	0.66	0.00
$^{228}\text{Th}^*$	1,2	4	0.34	0.34	0.66	0.66
$^{224}\text{Ra}^*$	1,2	5	0.34	0.34	0.66	0.66
$^{220}\text{Rn}^*$	1,2	6	0.34	0.34	0.66	0.66
$^{216}\text{Po}^*$	1,2	7	0.34	0.34	0.66	0.66
^{212}Pb	1,2	8	0.34	0.00	0.66	0.00
$^{212}\text{Bi}^*$	1	9	0.34	0.34	0.00	0.00
$^{212}\text{Bi}^*$	2	9	0.00	0.00	0.66	0.00
^{208}Tl	1	10	0.34	0.00	0.00	0.00
$^{212}\text{Po}^*$	2	10	0.00	0.00	0.66	0.66
^{208}Pb	1,2	--	--	--	--	--
Totals			3.40	2.04	6.60	3.96

Table A.8-1 shows clearly that, for this case, $N(1) = N(2) = 10$, $B(1) = 3.74$, $B_\alpha(1) = 2.04$, $B(2) = 7.26$, $B_\alpha(2) = 3.96$. The total activity for particle size d is given by $A(d)$, and the total alpha activity for particle size d is given by $A_\alpha(d)$, while the ALI in αdps is given by

$$ALI_\alpha = \frac{\text{Dose Limit} \cdot \sum_d A_\alpha(d) \left(\sum_{i=1}^2 \frac{B_\alpha(i)}{B(i)} F(i) \right)}{\sum_d A_\alpha(d) \sum_i \left(\frac{F(i)}{B(i)} \right) \sum_{j=1}^{N(i)} D(i,j,d) \beta(i,j)} \quad (\text{A.8-2})$$

The activity fractions for the two chains are given by $F(1) = 0.34$ and $F(2) = 0.66$. Thus the last equation expands to

$$ALI_\alpha(^{232}\text{Th}) = \frac{\text{Dose Limit} \cdot \sum_d A_\alpha(d) \left(\frac{B_\alpha(1)}{B(1)} F(1) + \frac{B_\alpha(2)}{B(2)} F(2) \right)}{\sum_d A_\alpha(d) \left(\left(\frac{F(1)}{B(1)} \right) \sum_{j=1}^{N(1)} D(1,j,d) \beta(1,j) + \left(\frac{F(2)}{B(2)} \right) \sum_{j=1}^{N(2)} D(2,j,d) \beta(2,j) \right)} \quad (\text{A.8-3})$$

In addition, $F(1)/B(1) = F(2)/B(2) (= 1/10)$. Therefore

$$ALI_{\alpha}(^{232}\text{Th}) = \frac{\text{Dose Limit} \cdot \sum_d A_{\alpha}(d)(B_{\alpha}(1) + B_{\alpha}(2))}{\sum_d A_{\alpha}(d) \left(\sum_{j=1}^{N(1)} D(1,j,d)\beta(1,j) + \sum_{j=1}^{N(2)} D(2,j,d)\beta(2,j) \right)} \quad (\text{A.8-4})$$

or

$$ALI_{\alpha}(^{232}\text{Th}) = \frac{\text{Dose Limit} \cdot \sum_d A_{\alpha}(d) \left(\sum_{j=1}^{N(1)} \beta_{\alpha}(1,j) + \sum_{j=1}^{N(2)} \beta_{\alpha}(2,j) \right)}{\sum_d A_{\alpha}(d) \left(\sum_{j=1}^{N(1)} D(1,j,d)\beta(1,j) + \sum_{j=1}^{N(2)} D(2,j,d)\beta(2,j) \right)} \quad (\text{A.8-5})$$

From Table A.8-1, it is obvious that

$$\sum_{j=1}^{N(1)} \beta_{\alpha}(1,j) + \sum_{j=1}^{N(2)} \beta_{\alpha}(2,j) = 2.04 + 3.96 = 6.0 \quad (\text{A.8-6})$$

In addition, $D(1,j,d) = D(2,j,d)$ for $j = 1$ to 9 . Therefore

$$\begin{aligned} & \sum_{j=1}^{N(1)} D(1,j,d)\beta(1,j) + \sum_{j=1}^{N(2)} D(2,j,d)\beta(2,j) \\ &= \sum_{j=1}^9 D(1,j,d)(\beta(1,j) + \beta(2,j)) + D(1,10,d)\beta(1,10) + D(2,10,d)\beta(2,10) \end{aligned} \quad (\text{A.8-7})$$

Table A.8-1 also shows that $\beta(1,j) + \beta(2,j) = 1$ for $j = 1$ to 9 . Therefore

$$\begin{aligned} & \sum_{j=1}^{N(1)} D(1,j,d)\beta(1,j) + \sum_{j=1}^{N(2)} D(2,j,d)\beta(2,j) \\ &= \sum_{j=1}^9 D(1,j,d) + D(1,10,d)\beta(1,10) + D(2,10,d)\beta(2,10) \end{aligned} \quad (\text{A.8-8})$$

Therefore

$$ALI_{\alpha}(^{232}\text{Th}) = \frac{\text{Dose Limit} \times 6.0 \times \sum_d A_{\alpha}(d)}{\sum_d A_{\alpha}(d) \left(\sum_{j=1}^9 D(1,j,d) + D(2,10,d)\beta(2,10) + D(2,11,d)\beta(2,11) \right)} \quad (\text{A.8-9})$$

The alternative approach is to treat the ^{232}Th series as a single series as shown in Table A.8-2.

Table A.8-2. Equilibrium factors for the Thorium chain.

Nuclide	i	j	$\beta(2,j)$	$\beta_{\alpha}(2,j)$
$^{232}\text{Th}^*$	2	1	1.00	1.00
^{228}Ra	2	2	1.00	0.00
^{228}Ac	2	3	1.00	0.00
$^{228}\text{Th}^*$	2	4	1.00	1.00
$^{224}\text{Ra}^*$	2	5	1.00	1.00
$^{220}\text{Rn}^*$	2	6	1.00	1.00
$^{216}\text{Po}^*$	2	7	1.00	1.00
^{212}Pb	2	8	1.00	0.00
$^{212}\text{Bi}^*$	2	9	1.00	0.34
^{208}Tl	2	10	0.34	0.00
$^{212}\text{Po}^*$	2	11	0.66	0.66
^{208}Pb	1,2	--	--	--
Totals			10.00	6.00

Therefore, in this case $N(2) = 11$, $B(2) = 10.00$, $B_{\alpha}(2) = 6.00$, so that the ALI is given by

$$ALI_{\alpha}(^{232}\text{Th}) = \frac{\text{Dose Limit} \cdot \sum_d A_{\alpha}(d) \left(\frac{B_{\alpha}(2)}{B(2)} F(2) \right)}{\sum_d A_{\alpha}(d) \left(\frac{F(2)}{B(2)} \right)^{N(2)} \sum_{j=1}^{N(2)} D(2,j,d)\beta(2,j)} \quad (\text{A.8-10})$$

Since $F(2) = 1$, and $\beta(2,j) = 1$ for $j = 1$ to 9 , this becomes

$$ALI_{\alpha}(^{232}\text{Th}) = \frac{\text{Dose Limit} \cdot \sum_d A_{\alpha}(d) B_{\alpha}(2)}{\sum_d A_{\alpha}(d) \left(\sum_{j=1}^9 D(2,j,d) + D(2,10,d)\beta(2,10) + D(2,11,d)\beta(2,11) \right)} \quad (\text{A.8-11})$$

which gives the same answer as before. This demonstrates that a chain with branches can be considered as a simple linear chain provided the nuclides are labelled carefully and the total equilibrium factors are calculated carefully.